Received: *July 17, 2001,* Accepted: *August 8, 2001*

Superstring BRST cohomology

Friedemann Brandt

Max-Planck-Institut für Mathematik in den Naturwissenschaften
Inselstraße 22-26, D-04103 Leipzig, Germany and
Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Am Mühlenberg 1, D-14476 Golm, Germany
E-mail: fbrandt@mis.mpg.de

Alexander Kling and Maximilian Kreuzer

Institut für Theoretische Physik, Technische Universität Wien
Wiedner Hauptstraße 8-10, A-1040 Vienna, Austria
E-mail: kling@hep.itp.tuwien.ac.at, kreuzer@hep.itp.tuwien.ac.at

Abstract: We first derive all world-sheet action functionals for NSR superstring models with (1,1) supersymmetry and any number of abelian gauge fields, for gauge transformations of the standard form. Then we prove for these models that the BRST cohomology groups $H^g(s)$, $g < 4$ (with the antifields taken into account) are isomorphic to those of the corresponding bosonic string models, whose cohomology is fully known. This implies that the nontrivial global symmetries, Noether currents, background charges, consistent deformations and candidate gauge anomalies of an NSR (1,1) superstring model are in one-to-one correspondence with their bosonic counterparts.

Keywords: BRST Quantization, Superstrings and Heterotic Strings, BRST Symmetry.

JHEP08(2001)023

Contents

1. Introduction and conclusion

We present in this paper a BRST cohomological analysis of superstring models in the NSR formulation [1, 2, 3] with local (1,1) supersymmetry [4, 5]. The class of models under study is quite general since it is characterized only by requirements on the field content and the gauge symmetries. The field content is given by the component fields of three types of supersymmetry multiplets: the 2d supergravity multiplet, 'matter multiplets' containing the 'target space coordinates', and abelian gauge field multiplets. The number of matter multiplets and gauge field multiplets is not fixed, i.e., our results apply to any target space dimension (1,2, ...) and an arbitrary number (0,1, ...) of abelian world-sheet gauge fields. The supersymmetry transformations are obtained from an analysis of the Bianchi identities of 2d supergravity in presence of abelian gauge fields.

The first part of our analysis is the determination of all local world-sheet actions compatible with these requirements, using the standard form of the gauge transformations (the question whether or not these transformations can be nontrivially deformed is a matter covered by the investigation in the second part of the paper, cf. comments at the end of section 4.1). This analysis is accomplished by a cohomological computation in the space of local functions which do not depend on antifields (this is possible because we use a formulation in which the commutator algebra of the gauge transformations closes off-shell). Its result has been reported and discussed already in [6]: when abelian gauge fields are absent, the cohomological analysis reproduces the general superstring action found already in [7]; in presence of abelian gauge fields, it yields locally supersymmetric extensions of the purely bosonic actions derived in [8, 9] and may be interpreted in terms of an enlarged target space with one 'frozen' extra dimension for each gauge field. In particular there are locally supersymmetric actions of the Born-Infeld type among these actions [6].

In the second part of the paper we analyse the local BRST cohomology $H(s)$ for the models whose world-sheet actions were determined in the first part.[1] Here and throughout this paper $H(s)$ denotes the cohomology of the BRST differential in the space of local functions which neither depend explicitly on the world-sheet coordinates nor on the world-sheet differentials, but only on the fields, antifields and their derivatives. This cohomology is the most important one for the models under study because the other local BRST cohomology groups can be easily derived from it. This is due to the invariance of the models under world-sheet diffeomorphisms, owing to a general property of diffeomorphism invariant theories discussed in detail in [11, sections 5 and 6] (see also [12]–[14]).

[1]The action is needed to fix the BRST transformations of the antifields. s denotes the BRST differential in the jet space associated with the fields and antifields [10]. Our analysis is general except for a very mild assumption (invertibility) on the "target space metric", see section 6.

JHEP08(2001)023

In particular, $H(s)$ yields directly the cohomology in form-degree 2 of s modulo the "world-sheet exterior derivative" d.[2] This cohomology is the most relevant one for physical applications and denoted by $H^{g,2}(s|d)$, where g specifies the ghost number sector. Cocycles of $H^{g,2}(s|d)$ are denoted by $\omega^{g,2}$ and the cocycle condition is

$$s\omega^{g,2} + d\omega^{g+1,1} = 0\,, \tag{1.1}$$

where $\omega^{g+1,1}$ is some local 1-form with ghost number $g+1$. $\omega^{g,2}$ is a coboundary in $H^{g,2}(s|d)$ if $\omega^{g,2} = s\omega^{g-1,2} + d\omega^{g,1}$ for some local forms $\omega^{g-1,2}$ and $\omega^{g,1}$. $H^{g,2}(s|d)$ is related to $H(s)$ through the descent equations as explained in [11]–[14]. The physically interesting cohomology groups $H^{g,2}(s|d)$ are those with ghost numbers $g < 2$: $H^{-1,2}(s|d)$ yields the nontrivial Noether currents and global symmetries [15], $H^{0,2}(s|d)$ and $H^{1,2}(s|d)$ determine the consistent deformations [16], background charges [17] and candidate gauge anomalies (see, e.g., [18]). The corresponding cohomology groups of s are $H^g(s)$ with $g < 4$. They are the objects of our second main result: we shall prove that these cohomology groups are isomorphic to their counterparts in the corresponding bosonic string models[3] [the bosonic model corresponding to a particular superstring model is obtained from the latter simply by setting all fermions to zero in the world-sheet action]. Furthermore, the correspondence is very explicit: the representatives of the s-cohomology of a superstring model are simply extensions of their "bosonic" counterparts, i.e. they contain the representatives of the s-cohomology of the corresponding bosonic string model and complete them to s-cocycles of the superstring model [analogously to the superstring action itself, which contains the bosonic string action and completes it to a locally supersymmetric one].

This result provides a complete characterization of the cohomology groups $H^g(s)$, $g < 4$ because the cohomology $H(s)$ for the bosonic string models has been completely determined in [19] (ordinary bosonic strings) and [9] (bosonic strings with world-sheet gauge fields). Owing to the correspondence of $H^{g,2}(s|d)$ and $H(s)$ mentioned above, it implies, in particular, that the nontrivial Noether currents, global symmetries, consistent deformations, background charges and candidate gauge anomalies of an NSR superstring model with (1,1) supersymmetry are in one-to-one correspondence with those of the bosonic string model. The results for the bosonic models were derived and discussed in detail in [19, 17, 8, 9, 20]. We shall not repeat or summarize them here.

We find the result quite remarkable and surprising since it means that the local (1,1) supersymmetry of the models under study has no effect on the structure of the cohomology at all. We note that our analysis and result applies analogously

[2]Actually d is defined on the jet space of the fields and antifields [10].

[3]We believe that the isomorphism extends to all higher ghost number sectors as well since most parts of our proof (in fact, everything except for the case-by-case study in appendix B) hold for all ghost numbers.

to heterotic strings with local (1,0) supersymmetry (by switching off one of the supersymmetries). However, we do not expect that it extends to superstrings with two or more local supersymmetries of the same chirality, such as heterotic strings with local (2,0) supersymmetry. These supersymmetries restrict already the world-sheet action to special backgrounds [21, 22, 23]. Accordingly, we expect that the local BRST cohomology of such superstring models is "smaller" than the one for corresponding bosonic strings.

The paper is organized as follows. In section 2 we specify the field content and the gauge and BRST transformations of the fields. In section 3 we construct field variables (jet space coordinates) which are well suited for the cohomological analysis. This involves the super-Beltrami parametrization for the gravitational multiplet and a construction of superconformal tensor fields for the matter and gauge multiplets. In section 4 we determine the most general action for the field content and gauge transformations introduced before by computing $H^2(s)$ in the space of antifield independent local functions. This completes the first part of our analysis. In section 5 we introduce the antifields, give their BRST transformations and extend the superconformal tensor calculus by constructing superconformal antifield variables. The next two sections contain the derivation of our second result: in section 6 we define and analyse an on-shell BRST cohomology $H(\sigma)$; in section 7 we show that $H^g(\sigma)$ is isomorphic to $H^g(s)$ and to the cohomology of the corresponding bosonic string model when $g < 4$. Some details of the analysis of sections 6 and 7 are collected in the appendices A and B. The remaining appendices give a short summary of the derivation of the gauge transformations from the supergravity Bianchi identities and contain a collection of the s-transformations of the covariant (= superconformal) field and antifield variables.

2. Field content and gauge symmetries

The field content of the models we are going to study is given by the supergravity multiplet consisting of the vielbein $e_m^{\ a}$, the gravitino χ_m^α and an auxiliary scalar field S.[4] Furthermore we consider a set of scalar multipets $\{X^M, \psi_\alpha^M, F^M\}$ corresponding to the string "target space coordinates" and their superpartners and a set of abelian gauge multiplets $\{A_m^i, \lambda_\alpha^i, \phi^i\}$. On minkowskian world-sheets all fields are real and the fermions are Majorana-Weyl spinors. The number of scalar multiplets and gauge multiplets is not specified, i.e. our approach covers any number of such fields. As gauge symmetries we impose world-sheet diffeomorphisms, local $2d$ Lorentz transformations, Weyl and super-Weyl transformations and of course local (1,1) world-sheet supersymmetry. Furthermore we require invariance under abelian gauge transformations of the A_m^i and under arbitrary local shifts of the auxiliary field S. The gauge

[4] m, a, α denote $2d$ world-sheet, Lorentz and spinor indices, respectively.

symmetries entail the corresponding ghost fields, which fixes the field content to

$$\Phi^A = \{e_m{}^a, \chi_m{}^\alpha, S, X^M, \psi^M_\alpha, F^M, A^i_m, \lambda^i_\alpha, \phi^i, \xi^m, \xi^\alpha, C^{ab}, C^W, \eta^\alpha, W, c^i\}\,,$$

where ξ^m denote the world sheet diffeomorphism ghosts, ξ^α are the supersymmetry ghosts and C^{ab} is the Lorentz ghost. C^W and η^α are the Weyl and super-Weyl ghosts, respectively. c^i are the ghosts associated with the U(1) transformations of the gauge fields and W denotes the ghost corresponding to the local shifts of the auxiliary field S. The gauge transformation of the supergravity multiplet written as BRST transformations are

$$\begin{aligned}
se_m{}^a &= \xi^n\partial_n e_m{}^a + (\partial_m\xi^n)e_n{}^a - 2\mathrm{i}\xi^\alpha\chi_m{}^\beta(\gamma^a C)_{\alpha\beta} + C_b{}^a e_m{}^b + C^W e_m{}^a \\
s\chi_m{}^\alpha &= \xi^n\partial_n\chi_m{}^\alpha + (\partial_m\xi^n)\chi_n{}^\alpha + \nabla_m\xi^\alpha - \frac{1}{4}\xi^\beta e_m{}^a S(\gamma_a)_\beta{}^\alpha + \frac{1}{2}C^W\chi_m{}^\alpha + \\
&\quad + \mathrm{i}\eta^\beta(\gamma_m)_\beta{}^\alpha - \frac{1}{4}C^{ab}\chi_m{}^\beta\varepsilon_{ab}(\gamma_*)_\beta{}^\alpha \\
sS &= \xi^n\partial_n S - 4\xi^\gamma(\gamma_* C)_{\gamma\alpha}\varepsilon^{nm}\nabla_n\chi_m{}^\alpha + \mathrm{i}\,\xi^\gamma(\gamma^m C)_{\gamma\alpha}\chi_m{}^\alpha S - C^W S + W\,, \qquad (2.1)
\end{aligned}$$

where $C_{\alpha\beta}$ is the charge conjugation matrix satisfying $-(\gamma^a)^T = C^{-1}(\gamma^a)C$. γ_* is defined through $\gamma^a\gamma^b = \eta^{ab}\mathbb{1} + \varepsilon^{ab}\gamma_*$ and $\varepsilon^{01} = \varepsilon_{10} = 1$. ∇_m denotes the Lorentz covariant derivative

$$\nabla_m = \partial_m - \frac{1}{2}\omega_m{}^{ab}l_{ab}$$

in terms of the Lorentz generator l_{ab} and the spin connection

$$\begin{aligned}
\omega_m{}^{ab} &= E^{an}E^{bk}(\omega_{[mn]k} - \omega_{[nk]m} + \omega_{[km]n}) \\
\omega_{[mn]k} &= e_{kd}\partial_{[n}e_{m]}{}^d - \mathrm{i}\chi_n\gamma_k\chi_m\,, \qquad E_a{}^m e_m{}^b = \delta_a{}^b\,. \qquad (2.2)
\end{aligned}$$

The BRST transformations of the scalar multiplets read

$$\begin{aligned}
sX^M &= \xi^m\partial_m X^M + \xi^\alpha\psi^M_\alpha \\
s\psi^M_\alpha &= \xi^m\partial_m\psi^M_\alpha - \mathrm{i}\xi^\beta(\gamma^m C)_{\beta\alpha}(\partial_m X^M - \chi_m{}^\gamma\psi^M_\gamma) + \xi^\beta C_{\beta\alpha}F^M + \\
&\quad + \frac{1}{4}C^{ab}\varepsilon_{ab}(\gamma_*)_\alpha{}^\beta\psi^M_\beta - \frac{1}{2}C^W\psi^M_\alpha \\
sF^M &= \xi^m\partial_m F^M + \xi^\alpha(\gamma^m)_\alpha{}^\beta\{\nabla_m\psi^M_\beta + \mathrm{i}\chi_m{}^\gamma(\gamma^n C)_{\gamma\beta}(\partial_n X^M - \chi_n{}^\delta\psi^M_\delta) - \\
&\qquad - \chi_m{}^\gamma C_{\gamma\beta}F^M\} - C^W F^M\,. \qquad (2.3)
\end{aligned}$$

The BRST transformations of the U(1) multiplets are

$$\begin{aligned}
s\phi^i &= \xi^n\partial_n\phi^i + \xi^\alpha(\gamma_*)_\alpha{}^\beta\lambda^i_\beta - C^W\phi^i \\
s\lambda^i_\beta &= \xi^n\partial_n\lambda^i_\beta + \xi^\alpha\big(\mathrm{i}(\gamma_* C)_{\alpha\beta}\varepsilon^{mn}(\partial_m A^i_n + \chi_m\gamma_n\lambda^i - \mathrm{i}\chi_n\gamma_* C\chi_m\phi^i) - \\
&\qquad - \mathrm{i}(\gamma_*\gamma^m C)_{\alpha\beta}(\partial_m\phi^i - \chi_m\gamma_*\lambda^i) + \mathrm{i}(\gamma_* C)_{\alpha\beta}S\phi^i\big) + \\
&\quad + \frac{1}{4}C^{ab}\varepsilon_{ab}(\gamma_*)_\beta{}^\gamma\lambda^i_\gamma + 2\eta^\alpha(\gamma_* C)_{\alpha\beta}\phi^i - \frac{3}{2}C^W\lambda^i_\beta \\
sA^i_m &= \xi^n\partial_n A^i_m + (\partial_m\xi^n)A^i_n + \partial_m c^i - 2\mathrm{i}\,\xi^\alpha\chi_m{}^\beta(\gamma_* C)_{\beta\alpha}\phi^i - \xi^\alpha(\gamma_m)_\alpha{}^\beta\lambda^i_\beta\,. \qquad (2.4)
\end{aligned}$$

These transformations were obtained by analyzing the $2d$ supergravity algebra in presence of the scalar matter and gauge multiplets [24] analogously to the superspace analysis of [25]. A short summary of the analysis is given in appendix C. In the supergravity sector we used the constraints

$$T_{\alpha\beta}{}^{a} = 2\mathrm{i}(\gamma^a C)_{\alpha\beta}, \quad T_{ab}{}^{c} = T_{\alpha\beta}{}^{\gamma} = 0 \tag{2.5}$$

and in the U(1) sector

$$F^i_{\alpha\beta} = 2\mathrm{i}(\gamma_* C)_{\alpha\beta}\phi^i\,. \tag{2.6}$$

All constraints are conventional, i.e., can be achieved by redefinitions of the connections. The transformations of the ghosts are such that the BRST differential s squares to zero,

$$\begin{aligned}
s\xi^n &= \xi^m\partial_m\xi^n + \mathrm{i}\xi^\alpha\xi^\beta(\gamma^n C)_{\alpha\beta} \\
s\xi^\alpha &= \xi^n\partial_n\xi^\alpha - \mathrm{i}\xi^\gamma\xi^\beta(\gamma^m C)_{\beta\gamma}\chi_m{}^\alpha - \frac{1}{4}C^{ab}\xi^\beta\varepsilon_{ab}(\gamma_*)_\beta{}^\alpha + \frac{1}{2}C^W\xi^\alpha \\
sC^{ab} &= \xi^m\partial_m C^{ab} - \frac{\mathrm{i}}{4}\xi^\alpha\xi^\beta S(\gamma_* C)_{\alpha\beta}\varepsilon^{ab} - \mathrm{i}\xi^\alpha\xi^\beta(\gamma^m C)_{\beta\alpha}\omega_m{}^{ab} - 2\eta^\beta\xi^\alpha(\gamma_* C)_{\alpha\beta}\varepsilon^{ab} \\
sC^W &= \xi^n\partial_n C^W + 2\eta^\beta\xi_\beta \\
s\eta^\alpha &= \xi^n\partial_n\eta^\alpha - \frac{1}{4}C^{ab}\eta^\beta\varepsilon_{ab}(\gamma_*)_\beta{}^\alpha + \mathrm{i}\xi^\beta(\gamma^n)_\beta{}^\alpha\left(\frac{1}{2}\partial_n C^W - \eta^\gamma(\chi_n C)_\gamma\right) - \\
&\quad -\frac{1}{2}C^W\eta^\alpha + \xi^\alpha W \\
sW &= \xi^n\partial_n W - 4\mathrm{i}\xi^\beta(\gamma^m C)_{\beta\alpha}\left(\nabla_m\eta^\alpha - \frac{1}{4}\chi_m{}^\alpha W - \frac{i}{2}\chi_m{}^\gamma(\gamma^n)_\gamma{}^\alpha(\partial_n C^W)\right) - \\
&\quad -4\xi^\beta\chi_m{}^\alpha(\gamma^m\gamma^n C)_{\alpha\beta}\eta^\gamma(\chi_n C)_\gamma - C^W W \\
sc^i &= \xi^m\partial_m c^i + \mathrm{i}\xi^\alpha\xi^\beta(\gamma_* C)_{\alpha\beta}\phi^i - \mathrm{i}\xi^\alpha\xi^\beta(\gamma^m C)_{\alpha\beta}A^i_m\,.
\end{aligned} \tag{2.7}$$

We remark that the use of Weyl, super-Weyl and Lorentz transformations, as well as the shift symmetry associated with the auxiliary field S are artefacts of the formulation and disappear in an equivalent formulation based on a Beltrami parametrization of the world-sheet zweibein (see sections 3 and 4). Of course we could have used the Beltrami approach from the very beginning, but we decided to start from the more familiar formulation presented above.

3. Superconformal tensor calculus

The first part of our cohomological analysis consists in the construction of a suitable "basis" for the fields and their derivatives (more precisely: suitable coordinates of the jet space associated with the fields). The goal is to find a basis $\{u^\ell, v^\ell, w^I\}$ with

JHEP08(2001)023

as many s-doublets (u^ℓ, v^ℓ) as possible and complementary (local) variables w^I such that sw^I can be expressed solely in terms of the w's, i.e.,

$$su^\ell = v^\ell, \quad sw^I = r^I(w)\,. \tag{3.1}$$

On general grounds, such a basis is related to a tensor calculus [14, 26, 27]. In the present case the tensor calculus is a superconformal one, generalizing the conformal tensor calculus in bosonic string models found in [19] (see also [9]). The w's with ghost number 1 are specific ghost variables corresponding to the superconformal algebra, the w's with ghost number 0 are "superconformal tensor fields" on which this algebra is represented.

3.1 Super-Beltrami parametrization

The superconformal structure of the models under consideration is related to the supersymmetric generalization of the so-called Beltrami parametrization [28, 29]. Beltrami differentials parametrize conformal classes of $2d$ metrics, and this makes them natural quantities to be used as basic variables in the present context. Since Beltrami differentials change only under world-sheet reparametrizations but not under Weyl or Lorentz transformations, their use leads to a simpler formulation of the models under study (cf. remarks at the end of section 2, and in section 4). In the following we choose a Euclidean notation and parametrize the worldsheet with independent variables z and $\bar z$ rather than with light cone coordinates, because this simplifies the notation and avoids some factors of i.[5]

As it is not hard to guess the supersymmetric generalization of the Beltrami parametrization involves in addition to the bosonic Beltrami differential μ a fermionic partner α, the Beltramino. The starting point is the parametrization of the vielbein

$$\begin{aligned} e^z &= (dz + d\bar z \mu_{\bar z}{}^z) e_z{}^z \\ e^{\bar z} &= (d\bar z + dz \mu_z{}^{\bar z}) e_{\bar z}{}^{\bar z}\,. \end{aligned} \tag{3.2}$$

The coefficients $\mu_{\bar z}{}^z$ and $\mu_z{}^{\bar z}$ are the Beltrami differentials

$$\begin{aligned} \mu &:= \mu_{\bar z}{}^z = \frac{e_{\bar z}{}^z}{e_z{}^z}\,, \\ \bar\mu &:= \mu_z{}^{\bar z} = \frac{e_z{}^{\bar z}}{e_{\bar z}{}^{\bar z}}\,, \end{aligned} \tag{3.3}$$

whereas the factors $e_z{}^z$ and $e_{\bar z}{}^{\bar z}$ are referred to as conformal factors. One should note that the Beltrami differentials transform under diffeomorphisms but do not change

[5] Note that reality conditions of spinors are subtle after Wick rotation to Euclidean space: In our left-right symmetric case of (1,1) supersymmetry we could define $(\psi)^* = \bar\psi$ and work with manifestly real actions, but obviously this would not be possible for heterotic theories. This is, however, irrelevant in our algebraic context.

JHEP08(2001)023

under Weyl or Lorentz transformations. The latter "structure group transformations" are carried solely by the conformal factors which form s-doublets (u^ℓ, v^ℓ) with ghost variables substituting (in the new basis) for the Lorentz ghost and the Weyl ghost.

The fermionic superpartners of the Beltrami differentials are suitable combinations of the gravitino fields

$$
\begin{aligned}
\alpha &:= \sqrt{\frac{8}{e_z{}^z}}\left(\chi_{\bar z}{}^2 - \mu\chi_z{}^2\right) \\
\bar\alpha &:= \sqrt{\frac{8}{e_{\bar z}{}^{\bar z}}}\left(\chi_z{}^1 - \bar\mu\chi_{\bar z}{}^1\right) \, .
\end{aligned} \tag{3.4}
$$

The Beltraminos are also invariant under structure group transformations. Especially they do not change under super-Weyl transformations. Again one can find complementary combinations of the gravitinos forming s-doublets with ghost variables that substitute for the super-Weyl ghosts. The fact that Weyl, Lorentz and super-Weyl ghosts (and not just their derivatives) occur in s-doublets as we just described reflects that Weyl, Lorentz and super-Weyl invariance are artefacts of the formulation.

The Beltrami parametrization involves also a redefinition of the diffeomorphism ghosts, sometimes called the Beltrami ghost fields. This again has to be supplemented with a redefinition of the supersymmetry ghosts. The new ghost variables, which replace the diffeomorphism ghosts ξ^z and $\xi^{\bar z}$ and the supersymmetry ghosts ξ^1 and ξ^2 are

$$
\begin{aligned}
\eta &:= (\xi^z + \mu\xi^{\bar z}) \\
\bar\eta &:= (\xi^{\bar z} + \bar\mu\xi^z) \\
\varepsilon &:= \frac{1}{2}(\hat\xi^2 + \xi^{\bar z}\alpha)\,, \qquad \hat\xi^2 := \sqrt{\frac{8}{e_z{}^z}}\xi^2 \\
\bar\varepsilon &:= \frac{1}{2}(\hat\xi^1 + \xi^z\bar\alpha)\,, \qquad \hat\xi^1 := \sqrt{\frac{8}{e_{\bar z}{}^{\bar z}}}\xi^1 \, .
\end{aligned} \tag{3.5}
$$

In terms of the new ghost variables the BRST transformations of "right-moving" and "left-moving" quantities decouple from each other [28],

$$
\begin{aligned}
s\mu &= \left(\bar\partial - \mu\partial + (\partial\mu)\right)\eta + \alpha\varepsilon \\
s\alpha &= \left(2\bar\partial - 2\mu\partial + (\partial\mu)\right)\varepsilon + \eta\partial\alpha + \frac{1}{2}\alpha\partial\eta \\
s\eta &= \eta\partial\eta - \varepsilon\varepsilon \\
s\varepsilon &= \eta\partial\varepsilon - \frac{1}{2}\varepsilon\partial\eta\,,
\end{aligned} \tag{3.6}
$$

with analogous transformations for the right movers.

3.2 Superconformal ghost variables and algebra

We have now paved the road for the construction of field variables $\{u^\ell, v^\ell, w^I\}$ fulfilling (3.1). In fact we have already identified some s-doublets (u^ℓ, v^ℓ), namely the u's given by the conformal factors and their fermionic counterparts and the corresponding v's given by ghost fields substituting in the new basis for the Weyl, Lorentz and super-Weyl ghosts. Furthermore, the field S obviously forms an s-doublet with a ghost field substituting for W. The derivatives of these u's and v's form s-doublets as well. The Beltrami differentials $\mu, \bar\mu$ and their derivatives are u's too. From (3.6) one observes that $s\mu$ and $s\bar\mu$ contain derivatives $\bar\partial\eta$ and $\partial\bar\eta$ and of the reparametrization ghosts, respectively. Taking derivatives of these transformations, one sees that the m-th derivatives of the Beltrami differentials pair off with ghost variables that substitute in the new basis for all $(m+1)$-th derivatives of the reparametrization ghosts except for $\partial^{m+1}\eta$ and $\bar\partial^{m+1}\bar\eta$. Analogously, the s-transformations of the Beltraminos contain derivatives $\bar\partial\varepsilon$ and $\partial\bar\varepsilon$ of the supersymmetry ghosts. Thus the m-th derivatives of α and $\bar\alpha$ pair off with ghost variables substituting for all $(m+1)$-th derivatives of ε and $\bar\varepsilon$ except for $\partial^{m+1}\varepsilon$ and $\bar\partial^{m+1}\bar\varepsilon$. We introduce the following notation for those ghost variables which do not sit in s-doublets:

$$\{C^N\} = \{\eta^p, \bar\eta^p, \varepsilon^{p+\frac{1}{2}}, \bar\varepsilon^{p+\frac{1}{2}} : p = -1, 0, 1, \dots\}, \tag{3.7}$$

with

$$\begin{aligned} \eta^p &= \frac{1}{(p+1)!}\,\partial^{p+1}\eta \\ \bar\eta^p &= \frac{1}{(p+1)!}\,\bar\partial^{p+1}\bar\eta \\ \varepsilon^{p+\frac{1}{2}} &= \frac{1}{(p+1)!}\,\partial^{p+1}\varepsilon \\ \bar\varepsilon^{p+\frac{1}{2}} &= \frac{1}{(p+1)!}\,\bar\partial^{p+1}\bar\varepsilon\,. \end{aligned} \tag{3.8}$$

These ghost variables fulfill the requirement imposed in (3.1) on w's. Indeed, using (3.6), one easily computes their s-transformations:

$$\begin{aligned} s\eta^p &= -\frac{1}{2}\eta^q\eta^r f_{rq}{}^p + \frac{1}{2}\varepsilon^a\varepsilon^b f_{ab}{}^p \\ &= \frac{1}{2}\eta^q\eta^r(r-q)\delta^p_{r+q} - \frac{1}{2}\varepsilon^a\varepsilon^b 2\delta^p_{a+b} \end{aligned} \tag{3.9}$$

$$\begin{aligned} s\varepsilon^a &= -\frac{1}{2}\eta^p\varepsilon^c f_{cp}{}^a + \frac{1}{2}\varepsilon^c\eta^p f_{pc}{}^a \\ &= -\varepsilon^c\eta^p\left(\frac{p}{2} - c\right)\delta^a_{p+c}\,. \end{aligned} \tag{3.10}$$

The f's which occur in these transformations are the structure constants of a graded commutator algebra of operators Δ_N to be represented on tensor fields constructed of the component fields of the matter and U(1) multiplets,

$$\{\Delta_N\} = \{L_p, \bar{L}_p, G_{p+\frac{1}{2}}, \bar{G}_{p+\frac{1}{2}} : p = -1, 0, 1, \dots\}\,. \tag{3.11}$$

This graded commutator algebra is nothing but the NS superconformal algebra

$$[L_p, L_q] = (p-q)L_{p+q}\,, \qquad \{G_a, G_b\} = 2L_{a+b}\,, \qquad [L_p, G_a] = \left(\frac{p}{2} - a\right) G_{p+a}\,, \tag{3.12}$$

with the analogous formulas for the $\bar{L}$'s and $\bar{G}$'s and the usual property that the holomorphic and antiholomorphic generators (anti-)commute,

$$\begin{array}{ll} [L_p, \bar{L}_q] = 0\,, & \{G_a, \bar{G}_b\} = 0\,, \\ [L_p, \bar{G}_a] = 0\,, & [\bar{L}_p, G_a] = 0\,. \end{array} \tag{3.13}$$

The representation of this algebra on superconformal tensor fields, and the explicit construction of these tensor fields, will be given in the following subsection.

3.3 Superconformal tensor fields

We shall now summarize the representation of the algebra (3.12) on superconformal tensor fields constructed of the fields and their derivatives (the representation on antifields is discussed in section 5) such that the BRST transformation of these tensor fields reads[6]

$$s\mathcal{T} = \sum_{p\geq -1} \left(\eta^p L_p + \bar{\eta}^p \bar{L}_p + \varepsilon^{p+\frac{1}{2}} G_{p+\frac{1}{2}} + \bar{\varepsilon}^{p+\frac{1}{2}} \bar{G}_{p+\frac{1}{2}}\right) \mathcal{T}\,. \tag{3.14}$$

The superconformal tensor fields corresponding to the fields X^M, ψ_α^M, F^M and their derivatives are denoted by $X^M_{m,n}$, $\psi^M_{m,n}$, $\bar{\psi}^M_{m,n}$, $F^M_{m,n}$ ($m, n \in \{0, 1, 2, \dots\}$). Here the subscripts m, n denote the number of operations L_{-1} and $\bar{L}_{-1}$ acting on $X^M_{0,0}$, $\psi^M_{0,0}$, $\bar{\psi}^M_{0,0}$, $F^M_{0,0}$, respectively (L_{-1} and $\bar{L}_{-1}$ will be identified with covariant derivatives, see below),

$$X^M_{0,0} \equiv X^M,\ \ \psi^M_{0,0} \equiv (e_z{}^z/2)^{\frac{1}{2}}\psi^M_2,\ \ \bar{\psi}^M_{0,0} \equiv (e_{\bar{z}}{}^{\bar{z}}/2)^{\frac{1}{2}}\psi^M_1,\ \ F^M_{0,0} \equiv \tfrac{1}{2}(e_z{}^z)^{\frac{1}{2}}(e_{\bar{z}}{}^{\bar{z}})^{\frac{1}{2}}F^M \tag{3.15}$$

$$X^M_{m,n} = (L_{-1})^m(\bar{L}_{-1})^n X^M_{0,0} \quad (m, n \in \{0, 1, 2, \dots\}) \quad \text{etc.} \tag{3.16}$$

The representation on these tensor fields can be inductively deduced from the algebra (3.12) using that all operations L_m, $\bar{L}_m$, G_a, $\bar{G}_a$ vanish on $X^M_{0,0}$ except for L_{-1}, $\bar{L}_{-1}$, $G_{-1/2}$ and $\bar{G}_{-1/2}$, with $G_{-1/2}X^M_{0,0} = \psi^M_{0,0}$ and $\bar{G}_{-1/2}X^M_{0,0} = \bar{\psi}^M_{0,0}$ (as can be read

[6] $\mathcal{T}$ stands for any of these superconformal tensor fields; η's and ε's are the ghost variables (3.8).

JHEP08(2001)023

off from sX^M). This gives on $X^M_{m,n}$:

$$
\begin{aligned}
L_p X^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p-1)!} X^M_{m-p,n} & \text{for } p < m \\ 0 & \text{for } p \geq m \end{cases} \\
\bar{L}_q X^M_{m,n} &= \begin{cases} \dfrac{n!}{(n-q-1)!} X^M_{m,n-q} & \text{for } q < n \\ 0 & \text{for } q \geq n \end{cases} \\
G_{p+\frac{1}{2}} X^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p-1)!} \psi^M_{m-p-1,n} & \text{for } p < m \\ 0 & \text{for } p \geq m \end{cases} \\
\bar{G}_{q+\frac{1}{2}} X^M_{m,n} &= \begin{cases} \dfrac{n!}{(n-q-1)!} \bar{\psi}^M_{m,n-q-1} & \text{for } q < n \\ 0 & \text{for } q \geq n \end{cases} .
\end{aligned}
$$

The action on the other fields is then easily obtained using

$$
[L_p, G_{-\frac{1}{2}}] = \frac{1}{2}(p+1)\, G_{p-\frac{1}{2}}\,, \qquad \{G_{p+\frac{1}{2}}, G_{-\frac{1}{2}}\} = 2L_p
$$

and the analogous formulas for $\bar{L}$ and $\bar{G}$ in (3.12). One obtains

$$
\begin{aligned}
L_p \psi^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p)!} \left(m - p + \dfrac{1}{2}(p+1) \right) \psi^M_{m-p,n} & \text{for } p \leq m \\ 0 & \text{for } p > m \end{cases} \\
G_{p+\frac{1}{2}} \psi^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p-1)!} X^M_{m-p,n} & \text{for } p < m \\ 0 & \text{for } p \geq m \end{cases} \\
\bar{G}_{q+\frac{1}{2}} \psi^M_{m,n} &= \begin{cases} -\dfrac{n!}{(n-q-1)!} F^M_{m,n-q-1} & \text{for } q < n \\ 0 & \text{for } q \geq n \end{cases} \\
\bar{L}_q \psi^M_{m,n} &= \begin{cases} \dfrac{n!}{(n-q-1)!} \psi^M_{m,n-q} & \text{for } q < n \\ 0 & \text{for } q \geq n \end{cases} \\
L_p F^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p)!} \left(m - p + \dfrac{1}{2}(p+1) \right) F^M_{m-p,n} & \text{for } p \leq m \\ 0 & \text{for } p > m \end{cases} \\
G_{p+\frac{1}{2}} F^M_{m,n} &= \begin{cases} \dfrac{m!}{(m-p-1)!}\, \bar{\psi}^M_{m-p,n} & \text{for } p < m \\ 0 & \text{for } p \geq m \end{cases}
\end{aligned}
$$

and analogous formulas for L's, G's, $\bar{L}$'s and $\bar{G}$'s acting on $\bar{\psi}^M_{m,n}$, and $\bar{L}$'s and $\bar{G}$'s acting on $F^M_{m,n}$.

The relation to the fields and their derivatives is established by identifying the operations L_{-1} and $\bar{L}_{-1}$ with covariant derivatives $\mathcal{D}$ and $\bar{\mathcal{D}}$ along the lines of [14],

$$
\begin{aligned}
L_{-1} \equiv \mathcal{D} = \frac{1}{1-\mu\bar{\mu}}\Big[\partial - \bar{\mu}\bar{\partial} - \sum_{p\geq 0}(\bar{M}^p\bar{L}_p - \bar{\mu}M^pL_p) - \sum_a(\bar{A}^a\bar{G}_a - \bar{\mu}A^aG_a)\Big] \\
\bar{L}_{-1} \equiv \bar{\mathcal{D}} = \frac{1}{1-\mu\bar{\mu}}\Big[\bar{\partial} - \mu\partial - \sum_{p\geq 0}(M^pL_p - \mu\bar{M}^p\bar{L}_p) - \sum_a(A^aG_a - \mu\bar{A}^a\bar{G}_a)\Big]
\end{aligned} \tag{3.17}
$$

where

$$
\begin{aligned}
M^p &= \frac{1}{(p+1)!}\partial^{p+1}\mu\,, \qquad \bar{M}^p = \frac{1}{(p+1)!}\bar{\partial}^{p+1}\bar{\mu}\,, \\
A^{p+\ 1/2} &= \frac{1}{(p+1)!2}\partial^{p+1}\alpha\,, \qquad \bar{A}^{p+\frac{1}{2}} = \frac{1}{(p+1)!2}\bar{\partial}^{p+1}\bar{\alpha}\,.
\end{aligned} \tag{3.18}
$$

One readily checks that these formulas result in local expressions for the superconformal tensor fields and their s-transformations. Introducing the following notation for the lowest weight superconformal matter fields

$$
X^M \equiv X^M_{0,0}\,, \qquad \psi^M \equiv \psi^M_{0,0}\,, \qquad \bar{\psi}^M \equiv \bar{\psi}^M_{0,0}\,, \qquad \hat{F}^M \equiv F^M_{0,0}\,, \tag{3.19}
$$

one gets in particular the following supercovariant derivatives

$$
\begin{aligned}
\mathcal{D}X^M &= \frac{1}{1-\mu\bar{\mu}}\Big[(\partial - \bar{\mu}\bar{\partial})X^M - \frac{1}{2}\bar{\alpha}\bar{\psi}^M + \frac{1}{2}\bar{\mu}\alpha\psi^M\Big] \\
\mathcal{D}\psi^M &= \frac{1}{1-\mu\bar{\mu}}\Big[(\partial - \bar{\mu}\bar{\partial})\psi^M + \frac{1}{2}\bar{\mu}(\partial\mu)\psi^M + \frac{1}{2}\bar{\alpha}\hat{F}^M + \frac{1}{2}\bar{\mu}\alpha\mathcal{D}X^M\Big] \\
\bar{\mathcal{D}}\psi^M &= \frac{1}{1-\mu\bar{\mu}}\Big[(\bar{\partial} - \mu\partial)\psi^M - \frac{1}{2}(\partial\mu)\psi^M - \frac{1}{2}\alpha\mathcal{D}X^M - \frac{1}{2}\mu\bar{\alpha}\hat{F}^M\Big]
\end{aligned} \tag{3.20}
$$

and analogous expressions for $\bar{\mathcal{D}}X^M$, $\bar{\mathcal{D}}\bar{\psi}^M$ and $\mathcal{D}\bar{\psi}^M$. We do not spell out higher order covariant derivatives explicitly because it turns out that they do not contribute nontrivially to the cohomology. The BRST transformations of the superconformal tensor fields are summarized in appendix D.

The construction of the superconformal tensor fields arising from the gauge multiplets is similar, once one has identified the suitable ghost variables and the lowest order tensor fields. The gauge fields A^i_m and their symmetrized derivatives $\partial_{(m_1}\dots\partial_{m_k}A^i_{m_{k+1})}$ ($k=1,2,\dots$) form s-doublets with ghost variables that substitute for all the derivatives of the ghosts c^i. Therefore one expects that only the undifferentiated ghosts c^i give rise to w-variables. Promising candidates for these w-variables are ghost variables C^i of the same form as in the purely bosonic case [9],

$$
C^i = c^i + \xi^m A^i_m\,. \tag{3.21}
$$

The s-transformations of the gauge fields, written in terms of C^i, and of the C^i themselves read

$$
\begin{aligned}
sA_m^i &= \xi^n(\partial_n A_m^i - \partial_m A_n^i) + \partial_m C^i - \xi^\alpha \chi_m{}^\beta F_{\alpha\beta}^i - \xi^\alpha e_m{}^a F_{a\alpha}^i \\
sC^i &= \xi^m \xi^n (\partial_m A_n^i - \partial_n A_m^i) + \frac{1}{2}\xi^\alpha \xi^\beta F_{\alpha\beta}^i + \xi^m \xi^\alpha \chi_m{}^\beta F_{\alpha\beta}^i + \xi^m \xi^\alpha F_{m\alpha}^i\,, \qquad (3.22)
\end{aligned}
$$

where we used notation of appendix C. Since we expect C^i to count among the w's, its s-transformation should involve only w's again, see (3.1). This suggests a strategy to determine the superconformal tensor fields corresponding to the undifferentiated fields ϕ^i, λ_α^i and to the field strengths of A_m^i: one tries to rewrite sC^i in (3.22) in terms of the ghost variables (3.8) and to read off from the result the sought superconformal tensor fields. This strategy turns out to be successful; one obtains

$$
sC^i = \eta\bar{\eta} F_{0,0}^i + \eta\bar{\varepsilon}\lambda_{0,0}^i + \bar{\eta}\varepsilon\bar{\lambda}_{0,0}^i + \varepsilon\bar{\varepsilon}\phi_{0,0}^i\,,
$$

where

$$
\begin{aligned}
\phi_{0,0}^i &= \sqrt{e_z{}^z e_{\bar{z}}{}^{\bar{z}}}\,\phi^i \\
\lambda_{0,0}^i &= \sqrt{\frac{e_{\bar{z}}{}^{\bar{z}}}{2}}\left(-e_z{}^z \lambda_2^i + \chi_z^2 \phi^i\right) \\
\bar{\lambda}_{0,0}^i &= \sqrt{\frac{e_z{}^z}{2}}\left(e_{\bar{z}}{}^{\bar{z}} \lambda_1^i + \chi_{\bar{z}}^1 \phi^i\right) \\
F_{0,0}^i &= \frac{1}{1-\mu\bar{\mu}}\left(\frac{1}{2}\varepsilon^{mn}(\partial_m A_n^i - \partial_n A_m^i) + \frac{1}{2}\mu\bar{\alpha}\lambda^i - \frac{1}{2}\bar{\mu}\alpha\bar{\lambda}^i - \frac{1}{4}\alpha\bar{\alpha}\phi^i\right). \qquad (3.23)
\end{aligned}
$$

An explicit computation shows that the s-transformations of these quantities are indeed of the desired form (3.14), with

$$
\lambda_{0,0}^i = G_{-\frac{1}{2}}\phi_{0,0}^i\,, \qquad \bar{\lambda}^i = \bar{G}_{-\frac{1}{2}}\phi_{0,0}^i\,, \qquad F_{0,0}^i = \bar{G}_{-\frac{1}{2}} G_{-\frac{1}{2}}\phi_{0,0}^i\,. \qquad (3.24)
$$

It is now straightforward to construct, along the previous lines, variables $\phi_{m,n}^i$, $\lambda_{m,n}^i$, $\bar{\lambda}_{m,n}^i$, $F_{m,n}^i$ on which the algebra (3.12) is represented and (3.14) and (3.17) hold. We do not spell out these tensor fields (with m or n different from 0) explicitly because it turns out that they do not contribute nontrivially to the cohomology. The resulting BRST transformations are summarized in appendix D too.

We introduce the following notation for the lowest order (i.e. lowest weight, see below) superconformal tensor fields arising from the gauge multiplet:

$$
\hat{\phi}^i \equiv \phi_{0,0}^i\,, \quad \lambda^i \equiv \lambda_{0,0}^i\,, \quad \bar{\lambda}^i \equiv \bar{\lambda}_{0,0}^i\,, \quad \mathcal{F}^i \equiv F_{0,0}^i\,. \qquad (3.25)
$$

Again tensor fields of higher order will be denoted by $\mathcal{D}\hat{\phi}^i$, $\bar{\mathcal{D}}\hat{\phi}^i$, $\mathcal{D}\bar{\mathcal{D}}\hat{\phi}^i$ etc. but as already stated above their explicit form will not be needed.

4. Action

We shall now determine the most general action for the field content and gauge transformations specified in section 2. The action has vanishing ghost number and is independent of antifields. Furthermore the requirement that the action be gauge invariant translates into BRST invariance up to surface terms. The integrands of the world-sheet actions we are looking for are thus the antifield independent solutions $\omega^{0,2}$ of equation (1.1). They are related through the descent equations to the solutions of

$$\begin{aligned} s\omega = 0\,, &\qquad \omega \neq s\hat{\omega}\,, \\ \mathrm{gh}\,(\omega) = 2\,, &\qquad \mathrm{agh}\,(\omega) = \mathrm{agh}\,(\hat{\omega}) = 0\,, \end{aligned} \tag{4.1}$$

where gh is the ghost number and agh is the antifield number (="antighost number", see section 5 for the definition). In the previous section we have constructed a basis for the fields and their derivatives satisfying the requirements of (3.1). By standard arguments this implies that ω and $\hat{\omega}$ can be assumed to depend only on the w^I, i.e., on superconformal tensor and ghost fields introduced in section 3.[7] Furthermore we can restrict the investigation to functions ω and $\hat{\omega}$ with vanishing "conformal weights" by an argument used already in [19, 9]: we extend the definition of L_0 and $\bar{L}_0$ to all w's (including the ghost variables) by

$$\Big\{ s\,,\, \frac{\partial}{\partial(\partial\eta)} \Big\} w^I = L_0\, w^I\,, \qquad \Big\{ s\,,\, \frac{\partial}{\partial(\bar{\partial}\bar{\eta})} \Big\} w^I = \bar{L}_0\, w^I\,. \tag{4.2}$$

Hence, in the space of local functions of the w's the derivatives with respect to $\partial\eta$ and $\bar{\partial}\bar{\eta}$ are contracting homotopies for L_0 and $\bar{L}_0$, respectively, and the cohomology can be nontrivial only in the intersection of the kernels of L_0 and $\bar{L}_0$.

All w's are eigenfunctions of L_0 and $\bar{L}_0$ with the eigenvalues being their "conformal weights". The only w^I with negative conformal weights are the undifferentiated diffeomorphism ghosts $\eta, \bar{\eta}$ and the undifferentiated supersymmetry ghosts $\varepsilon, \bar{\varepsilon}$; their conformal weights are $(-1, 0)$, $(0, -1)$, $(-1/2, 0)$ and $(0, -1/2)$, respectively [here (a, b) are the eigenvalues of $(L_0, \bar{L}_0)$]. The only superconformal tensor fields with vanishing conformal weights are the undifferentiated X^M. These properties simplify the analysis enormously.

Our strategy for finding the solutions to (4.1) will be based on an expansion in supersymmetry ghosts

$$\begin{aligned} \omega = \sum_{k=0}^{\bar{k}} \omega_k\,, &\qquad (N_\varepsilon + N_{\bar{\varepsilon}})\omega_k = k\omega_k \\ s = s_2 + s_1 + s_0\,, &\quad [N_\varepsilon + N_{\bar{\varepsilon}}, s_k] = k s_k\,, \end{aligned} \tag{4.3}$$

[7] The u's and v's contribute only "topologically" via the de Rham cohomology of the zweibein manifold to the s-cohomology, cf. [11, theorem 5.1]. In particular they do not contribute nontrivially to the solutions of (4.1).

where we have introduced the counting operator N_ε for the susy ghost ε and all its derivatives

$$N_\varepsilon = \sum_{n\geq 0}(\partial^n\varepsilon)\,\frac{\partial}{\partial(\partial^n\varepsilon)} \tag{4.4}$$

and analogously $N_{\bar\varepsilon}$ counts $\bar\varepsilon$ and derivatives thereof.[8] One observes that s_2 is the simplest piece in the above decomposition of s. It acts nontrivially only on the reparametrization ghosts η, $\bar\eta$, derivatives thereof and on C^i,

$$s_2\eta = -\varepsilon\varepsilon\,, \qquad s_2\bar\eta = -\bar\varepsilon\bar\varepsilon\,, \qquad s_2 C^i = \varepsilon\bar\varepsilon\hat\phi^i\,.$$

We shall base the investigation on the cohomology of s_2. The cocycle condition $s\omega = 0$ decomposes into

$$s_2\omega_{\bar k} = 0, \quad s_1\omega_{\bar k} + s_2\omega_{\bar k-1} = 0, \quad \ldots \tag{4.5}$$

Due to the requirement of ghost number 2 and antifield number 0 in (4.1), one is left with $0 \leq \bar k \leq 2$. The three possible values for $\bar k$ are now analysed case by case.

$\underline{\bar k=0}$: The general form of $\omega_{\bar 0}$ according to the condition of vanishing conformal weight is

$$\begin{aligned}\omega_{\bar 0} = {} & \eta\bar\eta A_{(1,1)} + \eta\partial\eta A_{(1,0)} + \bar\eta\bar\partial\bar\eta A_{(0,1)} + \eta\bar\partial\bar\eta B_{(1,0)} + \bar\eta\partial\eta B_{(0,1)} + \\ & +\eta\partial^2\eta A_{(0,0)} + \bar\eta\bar\partial^2\bar\eta\bar A_{(0,0)} + \partial\eta\bar\partial\bar\eta B_{(0,0)} + C^iC^jD_{ij(0,0)} + \\ & +\eta C^iD_{i(1,0)} + \bar\eta C^iD_{i(0,1)} + \partial\eta C^iD_{i(0,0)} + \bar\partial\bar\eta C^i\bar D_{i(0,0)}\,,\end{aligned}$$

where the A's, B's and D's do not depend on the ghosts and the subscripts (m,n) indicate their conformal weights. It is easy to verify explicitly that

$$s_2\omega_{\bar 0} = 0 \;\Leftrightarrow\; \omega_{\bar 0} = 0\,. \tag{4.6}$$

$\underline{\bar k=1.}$ The general form of $\omega_{\bar 1}$ is

$$\begin{aligned}\omega_{\bar 1} = {} & \eta\varepsilon A_{(3/2,0)} + \bar\eta\bar\varepsilon A_{(0,3/2)} + \eta\bar\varepsilon A_{(1,1/2)} + \bar\eta\varepsilon A_{(1/2,1)} + \\ & +\eta\partial\varepsilon A_{(1/2,0)} + \bar\eta\bar\partial\bar\varepsilon A_{(0,1/2)} + \varepsilon\partial\eta B_{(1/2,0)} + \bar\varepsilon\bar\partial\bar\eta B_{(0,1/2)} + \\ & +\varepsilon\bar\partial\bar\eta C_{(1/2,0)} + \bar\varepsilon\partial\eta C_{(0,1/2)} + \varepsilon C^iD_{i(1/2,0)} + \bar\varepsilon C^iD_{i(0,1/2)}\,,\end{aligned}$$

where again the A's, B's and D's do not depend on the ghosts and their conformal weights are indicated in brackets. A straightforward computation shows that $s_2\omega_{\bar 1} = 0$ imposes

$$\begin{aligned}A_{(3/2,0)} &= A_{(0,3/2)} = C_{(1/2,0)} = C_{(0,1/2)} = 0\\ A_{(1,1/2)} &= \hat\phi^iD_{i(1/2,0)}\,, \quad A_{(1/2,1)} = \hat\phi^iD_{i(0,1/2)}\\ A_{(1/2,0)} &= -2B_{(1/2,0)}\,, \quad A_{(0,1/2)} = -2B_{(0,1/2)}\,.\end{aligned}$$

[8]We note that the expansion (4.3) holds because we are studying the antifield independent cohomology here. The analogous expansion in presence of antifields is more involved; in fact, it can even involve infinitely many terms. Therefore the strategy applied here to determine the action is not practicable in the same way for analysing the full (antifield dependent) cohomology later.

The conformal weights $(1/2,0)$ and $(0,1/2)$ imply

$$D_{i(1/2,0)} = \psi^M D_{Mi}(X),\ \ D_{i(0,1/2)} = \bar\psi^M \bar D_{Mi}(X)$$
$$B_{(1/2,0)} = \psi^M B_M(X),\ \ B_{(0,1/2)} = \bar\psi^M \bar B_M(X)\,,$$

where we indicated that the remaining B's and D's are arbitrary functions of the X's. Hence, we get

$$\begin{aligned}\omega_{\bar 1} =\ & (\eta\bar\varepsilon\hat\phi^i + \varepsilon C^i)\psi^M D_{Mi}(X) + (\bar\eta\varepsilon\hat\phi^i + \bar\varepsilon C^i)\bar\psi^M \bar D_{Mi}(X) + \\ & + (\varepsilon\partial\eta - 2\eta\partial\varepsilon)\psi^M B_M(X) + (\bar\varepsilon\bar\partial\bar\eta - 2\bar\eta\bar\partial\bar\varepsilon)\bar\psi^M \bar B_M(X).\end{aligned}$$

The second equation (4.5) requires that $s_1\omega_{\bar 1}$ be s_2-exact. This imposes

$$\begin{aligned} & B_M = \bar B_M = 0, \quad D_{Mi} = \bar D_{Mi}\,, \quad \partial_N \bar D_{iM} = \partial_M D_{iN} \\ \Leftrightarrow\ & B_M = \bar B_M = 0, \quad D_{Mi} = \bar D_{Mi} = \partial_M D_i(X)\,,\end{aligned}$$

where we have introduced the notation

$$\partial_M = \frac{\partial}{\partial X^M}\,.$$

Furthermore, the second equation (4.5) uniquely determines the function ω_0, which corresponds to $\omega_{\bar 1}$ [the uniqueness follows from (4.6)]. It turns out that the other equations (4.5) do not impose further conditions in this case, but are automatically fulfilled. Altogether we find

$$\omega_{\bar 1} = [(\eta\bar\varepsilon\hat\phi^i + \varepsilon C^i)\psi^M + (\bar\eta\varepsilon\hat\phi^i + \bar\varepsilon C^i)\bar\psi^M]\partial_M D_i(X) \tag{4.7}$$
$$\begin{aligned}\omega_0 =\ & -\eta\bar\eta[\psi^M\bar\lambda^i - \bar\psi^M\lambda^i + \hat F^M\hat\phi^i + \psi^M\bar\psi^N\hat\phi^i\partial_N]\partial_M D_i(X) + \\ & + C^i(\eta\mathcal{D}X^M + \bar\eta\bar{\mathcal{D}}X^M)\partial_M D_i(X)\,.\end{aligned} \tag{4.8}$$

Using the freedom to add a coboundary we obtain by adding $s[C^i D_i(X)]$ to $\omega_{\bar 1} + \omega_0$ the equivalent solution

$$\begin{aligned} & \eta\bar\eta\mathcal{F}^i D_i(X) - \eta\bar\eta(\psi^M\bar\lambda^i - \bar\psi^M\lambda^i + \hat F^M\hat\phi^i + \psi^M\bar\psi^N\hat\phi^i\partial_N)\partial_M D_i(X) + \\ & + \eta\bar\varepsilon(\lambda^i + \hat\phi^i\psi^M\partial_M)D_i(X) + \bar\eta\varepsilon(\bar\lambda^i + \hat\phi^i\bar\psi^M\partial_M)D_i(X) + \varepsilon\bar\varepsilon\hat\phi^i D_i(X)\,.\end{aligned} \tag{4.9}$$

$\underline{\bar k{=}2.}$ The general form of $\omega_{\bar 2}$ is given by

$$\omega_{\bar 2} = \varepsilon\varepsilon A_{(1,0)} + \bar\varepsilon\bar\varepsilon A_{(0,1)} + \varepsilon\bar\varepsilon A_{(1/2,1/2)} + \varepsilon\partial\varepsilon B(X) + \bar\varepsilon\bar\partial\bar\varepsilon\bar B(X)\,,$$

where due to the indicated conformal weights one has

$$\begin{aligned} A_{(1,0)} &= \mathcal{D}X^M A_M(X) + \psi^M\psi^N A_{MN}(X) \\ A_{(0,1)} &= \bar{\mathcal{D}}X^M \bar A_M(X) + \bar\psi^M\bar\psi^N \bar A_{MN}(X) \\ A_{(1/2,1/2)} &= \hat F^M H_M(X) + \hat\phi^i H_i(X) + \psi^M\bar\psi^N H_{MN}(X)\,.\end{aligned}$$

We can simplify $\omega_{\bar{2}}$ using the freedom to subtract s-exact pieces from an s-cocycle. In particular, we can therefore neglect pieces in $\omega_{\bar{2}}$ which are of the form $s_1\hat{\omega}_1+s_2\hat{\omega}_0$ (i.e. we consider $\omega' = \omega - s(\hat{\omega}_1+\hat{\omega}_0)$ where ω is an s-cocycle arising from $\omega_{\bar{2}}$). Choosing

$$\hat{\omega}_1 = \frac{1}{2}\,(\bar{\varepsilon}\bar{\psi}^M - \varepsilon\psi^M)H_M(X)$$

we get

$$\begin{aligned} s_1\hat{\omega}_1 = {} & \varepsilon\bar{\varepsilon}\hat{F}^M H_M(X) + \frac{1}{2}(\bar{\varepsilon}\bar{\varepsilon}\bar{\mathcal{D}}X^M - \varepsilon\varepsilon\mathcal{D}X^M)H_M(X) \\ & -\frac{1}{2}(\bar{\varepsilon}\bar{\psi}^M - \varepsilon\psi^M)(\bar{\varepsilon}\bar{\psi}^N + \varepsilon\psi^N)\partial_N H_M(X)\,. \end{aligned}$$

This shows that by subtracting $s_1\hat{\omega}_1$ from $\omega_{\bar{2}}$, we can remove the piece $\hat{F}^M H_M(X)$ from $A_{(1/2,1/2)}$, thereby redefining $A_{(1,0)}$, $A_{(0,1)}$ and $H_{MN}(X)$. Furthermore, we have

$$\begin{aligned} & \varepsilon\varepsilon A_{(1,0)} + \bar{\varepsilon}\bar{\varepsilon}A_{(0,1)} + \varepsilon\bar{\varepsilon}\hat{\phi}^i H_i(X) + \varepsilon\partial\varepsilon B(X) + \bar{\varepsilon}\bar{\partial}\bar{\varepsilon}\bar{B}(X) = s_2\hat{\omega}_0\,, \\ & \hat{\omega}_0 = -\eta A_{(1,0)} - \bar{\eta}A_{(0,1)} + C^i H_i(X) - \frac{1}{2}\partial\eta B(X) - \frac{1}{2}\bar{\partial}\bar{\eta}\bar{B}(X)\,. \end{aligned}$$

Hence, we can also remove the pieces containing $A_{(1,0)}$, $A_{(0,1)}$, $H_i(X)$, $B(X)$ and $\bar{B}(X)$ from $\omega_{\bar{2}}$. Without loss of generality, we can thus restrict the investigation of the case $\bar{k}=2$ to

$$\omega_{\bar{2}} = \varepsilon\bar{\varepsilon}\psi^M\bar{\psi}^N H_{MN}(X)\,. \tag{4.10}$$

Obviously $\omega_{\bar{2}}$ satisfies the first eqation (4.5), since it does not involve η, $\bar{\eta}$ or C^i. One now has to analyze the remaining equations (4.5). It is straightforward to compute $s_1\omega_{\bar{2}}$ and to verify that the second equation (4.5) is solved by

$$\begin{aligned} \omega_1 = {} & \eta\bar{\varepsilon}[\mathcal{D}X^M\bar{\psi}^N - \psi^M\hat{F}^N + \psi^M\bar{\psi}^N\psi^K\partial_K]H_{MN}(X) + \\ & +\bar{\eta}\varepsilon[-\psi^M\bar{\mathcal{D}}X^N - \hat{F}^M\bar{\psi}^N + \psi^M\bar{\psi}^N\bar{\psi}^K\partial_K]H_{MN}(X)\,. \end{aligned} \tag{4.11}$$

The third eq. (4.5) requires that $s_0\omega_{\bar{2}} + s_1\omega_1$ be s_2-exact. This turns out to be the case (for arbitrary H_{MN}) and determines ω_0. One finds

$$\begin{aligned} \omega_0 = {} & \eta\bar{\eta}\Omega\,, \\ \Omega = {} & (\mathcal{D}X^M\bar{\mathcal{D}}X^N + \hat{F}^M\hat{F}^N + \mathcal{D}\bar{\psi}^M\bar{\psi}^N - \psi^M\bar{\mathcal{D}}\psi^N)H_{MN}(X) - \\ & -(\mathcal{D}X^M\bar{\psi}^N\bar{\psi}^K + \bar{\mathcal{D}}X^N\psi^M\psi^K)\partial_K H_{MN}(X) + \\ & +(\hat{F}^M\psi^K\bar{\psi}^N - \hat{F}^K\psi^M\bar{\psi}^N + \hat{F}^N\psi^M\bar{\psi}^K)\partial_K H_{MN}(X) + \\ & +\psi^M\psi^K\bar{\psi}^N\bar{\psi}^L\partial_K\partial_L H_{MN}(X)\,. \end{aligned} \tag{4.12}$$

The remaining two equations (4.5) are also satisfied. The functions $H_{MN}(X)$ are completely arbitrary. The symmetrized part $H_{(MN)}(X)$ and the antisymmetrized part $H_{[MN]}(X)$ give rise to the "target space metric" G_{MN} and the "Kalb-Ramond field" B_{MN}, respectively. Despite of our string inspired terminology we stress that there are no conditions imposed on G_{MN} and B_{MN} apart from their symmetry properties. In particular the "metric" G_{MN} need not be invertible (in section 6 we shall impose that a submatrix of G_{MN} be invertible). B_{MN} is determined only up to

$$H_{[MN]}(X) \to H_{[MN]}(X) + \partial_{[M} B_{N]}(X)\,,$$

where $B_M(X)$ are arbitrary functions. This originates from the fact that the s-cocycle $\omega = \omega_{\bar{2}} + \omega_1 + \omega_0$ remains form invariant under

$$\omega \to \omega + s\big[(\varepsilon\psi^M + \bar{\varepsilon}\bar{\psi}^M + \eta\mathcal{D}X^M + \bar{\eta}\bar{\mathcal{D}}X^M)B_M(X) + \ldots\big]\,,$$

where the dots stand for terms at least bilinear in the fermions. Changing ω by such s-exact pieces results in the above change of $H_{[MN]}(X)$ and modifies the Lagrangian by a total derivative.

4.1 Result

We conclude that up to redefinitions by coboundary terms, the general solution of (4.1) is given by the sum of the functions (4.9)–(4.12). The solution involves arbitrary functions $D_i(X)$ and $H_{MN}(X)$, which thus parametrize the various possible actions. The antisymmetric part of $H_{MN}(X)$ is determined only up to redefinitions by $\partial_{[M}B_{N]}(X)$, as $H_{MN}(X) \to H_{MN}(X) + \partial_{[M}B_{N]}(X)$ modifies the Lagrangian only by total derivatives. The functions $D_i(X)$ are determined up to arbitrary constants, since only $\partial_M D_i(X)$ enters in the equivalent solution (4.7) and (4.8).[9] Owing to general properties of descent equations in diffeomorphism invariant theories [11]–[14], the integrand of the action is obtained from the solution of (4.1) simply by substituting world-sheet differentials for diffeomorphism ghosts ξ^m. The resulting Lagrangian, written in terms of the Beltrami fields, is a generalized version of the

[9] A constant in D_i yields a topological term in the action proportional to the Chern class of the gauge bundle.

one found in [28]:

$$
\begin{aligned}
L &= L_{\text{Matter}} + L_{U1} \\
L_{\text{Matter}} &= \frac{1}{1-\mu\bar{\mu}}\Bigg[(\partial-\bar{\mu}\bar{\partial})X^M(\bar{\partial}-\mu\partial)X^N(G_{MN}+B_{MN}) - \\
&\qquad\qquad - \left((\partial-\bar{\mu}\bar{\partial})X^M\alpha\psi^N + (\bar{\partial}-\mu\partial)X^M\bar{\alpha}\bar{\psi}^N\right)G_{MN} - \\
&\qquad\qquad - \frac{1}{2}\alpha\bar{\alpha}\psi^M\bar{\psi}^N G_{MN}\Bigg] - \\
&\quad - \left(\bar{\psi}^N(\partial-\bar{\mu}\bar{\partial})\bar{\psi}^M + \psi^N(\bar{\partial}-\mu\partial)\psi^M\right)G_{MN} - (1-\mu\bar{\mu})\hat{F}^M\hat{F}^N G_{MN} - \\
&\quad - \bar{\psi}^M\bar{\psi}^N(\partial-\bar{\mu}\bar{\partial})X^K\left(\Gamma_{KNM}-\frac{1}{2}H_{KNM}\right) - \psi^M\psi^N(\bar{\partial}-\mu\partial)X^K \times \\
&\quad \times \left(\Gamma_{KNM}+\frac{1}{2}H_{KNM}\right) + \frac{1}{6}(\bar{\alpha}\bar{\psi}^M\bar{\psi}^N\bar{\psi}^K - \alpha\psi^M\psi^N\psi^K)H_{KMN} + \\
&\quad + (1-\mu\bar{\mu})\hat{F}^M\psi^K\bar{\psi}^N(2\Gamma_{KNM}-H_{KNM}) + \frac{1}{2}(1-\mu\bar{\mu})\psi^M\psi^K\bar{\psi}^N\bar{\psi}^L R_{KMLN} \\
L_{U1} &= F^i D_i - (1-\mu\bar{\mu})\Bigg[\psi^M\left(\bar{\lambda}^i - \frac{1}{2}\frac{1}{1-\mu\bar{\mu}}\mu\bar{\alpha}\hat{\phi}^i\right) - \\
&\qquad\qquad\qquad - \bar{\psi}^M\left(\lambda^i - \frac{1}{2}\frac{1}{1-\mu\bar{\mu}}\bar{\mu}\alpha\hat{\phi}^i\right) + \\
&\qquad\qquad\qquad + \hat{F}^M\hat{\phi}^i + \psi^M\bar{\psi}^N\partial_N\Bigg]\partial_M D_i\,, \qquad (4.13)
\end{aligned}
$$

where we have introduced the following notations

$$
\begin{aligned}
G_{MN} &:= H_{(MN)}(X) \qquad B_{MN} := H_{[MN]}(X) \\
D_i &:= D_i(X) \qquad F^i := \varepsilon^{mn}(\partial_m A^i_n - \partial_n A^i_m) \\
\Omega_{KNM} &:= \partial_K H_{MN}(X) - \partial_M H_{KN}(X) + \partial_N H_{KM}(X) = 2\Gamma_{KNM} - H_{KNM} \\
R_{KLMN} &:= \partial_M\partial_{[K}H_{L]N}(X) - \partial_N\partial_{[K}H_{L]M}(X)
\end{aligned}
$$

The "target space curvature" R_{KLMN} we have introduced is of course not the Riemannian one. The Riemannian curvature appears after eliminating the auxiliary fields from the action.

Of course, the action can be also written in terms of the original fields introduced in section 2. One obtains from the matter part the well known superstring action

JHEP08(2001)023

including the B-field background [7]

$$
\begin{aligned}
L/e = &\frac{1}{2}\partial_m X^M \partial_n X^N(-h^{mn}G_{MN} + \varepsilon^{mn}B_{MN}) + \frac{\mathrm{i}}{2}\overline{\psi}^M\gamma^m\partial_m\psi^N G_{MN} + \\
&+\frac{1}{2}F^M F^N G_{MN} + \chi_k\gamma^n\gamma^k\left(\psi^N\partial_n X^M - \frac{1}{4}C\chi_n\overline{\psi}^M\psi^N\right)G_{MN} + \\
&+\left(\frac{1}{2}F^M\overline{\psi}^K\psi^N - \mathrm{i}\,\overline{\psi}^N\gamma^m\psi^M\partial_m X^K\right)\Gamma_{NKM} + \\
&+\frac{1}{4}(F^M\overline{\psi}^K\gamma_*\psi^N - \mathrm{i}\overline{\psi}^N\gamma^m\gamma_*\psi^M\partial_m X^K)H_{NKM} - \\
&-\frac{\mathrm{i}}{12}\chi_m\gamma^n\gamma^m\psi^M\overline{\psi}^N\gamma_n\gamma_*\psi^K H_{MNK} + \\
&+\frac{1}{16}\overline{\psi}^M(\mathbb{1}+\gamma_*)\psi^N\overline{\psi}^K(\mathbb{1}+\gamma_*)\psi^L R_{KMLN} + \\
&+\varepsilon^{mn}D_i\partial_m A_n^i + \frac{\mathrm{i}}{4}\overline{\psi}^M\psi^N\phi^i\partial_N\partial_M D_i + \\
&+\frac{1}{2}(\mathrm{i}\overline{\psi}^N\gamma_*\lambda^i - \mathrm{i}F^N\phi^i + \chi_m\gamma^m\psi^N\phi^i)\partial_N D_i\,.
\end{aligned} \tag{4.14}
$$

Thus the cohomological analysis shows that in the absence of gauge multiplets the Lagrangian derived in [7] is in fact unique up to total derivatives and choices of the background fields. It should be kept in mind, however, that this uniqueness is tied to the gauge transformations specified in section 2. It can get lost when one allows that the gauge transformations get deformed. This deformation problem can be analysed by BRST cohomological means too, but then the relevant cohomological problem includes the antifields [16]. The results which we shall derive in the second part of this work imply that the nontrivial consistent deformations correspond one-to-one to the deformations of the bosonic string models. All deformations of bosonic string models without world-sheet gauge fields were derived in [17]. Hence, all nontrivial deformations of the standard superstring world-sheet action [7] and its gauge transformations are supersymmetric generalizations of the actions and gauge transformations given in [17]; in particular this result implies that nontrivial deformations of the standard superstring gauge transformations exist only if the background possesses special Killing vectors described in [17]. A full analysis (to all orders in the deformation parameters) of the deformation problem for bosonic models with world-sheet gauge fields is missing so far, but a complete classification of the first order deformations was given in [9]. The latter results extend thus to the superstring models too.

5. Antifields

To proceed with our analysis we have to bring the antifields into the game. According to the principles of the field-antifield formalism [30]–[33] to each field a corresponding antifield Φ^*_A is introduced with ghost number and statistics

$$\mathrm{gh}\,(\Phi^*_A) = -\mathrm{gh}\,(\Phi^A) - 1, \quad \epsilon(\Phi^*_A) = \epsilon(\Phi^A) + 1 \quad (\mathrm{mod}\ 2)\,,$$

such that the statistics of the antifields is opposite to that of the corresponding fields. It is useful to introduce still another grading into the algebra of fields and antifields, namely the already mentioned antifield (or antighost) number. On all the fields (including the ghosts) the antifield number is defined to be zero, i.e., $\mathrm{agh}\,(\Phi^A) = 0$. On the antifields the antifield number equals minus the ghost number, $\mathrm{agh}\,(\Phi^*_A) = -\mathrm{gh}\,(\Phi^*_A)$.

The antibracket for two arbitrary functions of the fields Φ^A and antifields Φ^*_A is defined as

$$(F, G) = \int \left(\frac{\delta_R F}{\delta \Phi^A} \frac{\delta_L G}{\delta \Phi^*_A} - \frac{\delta_R F}{\delta \Phi^*_A} \frac{\delta_L G}{\delta \Phi^A} \right).$$

Thus the antibracket has odd statistics and carries ghost number one. The BRST transformations of the antifields are generated via the antibracket by the proper solution $\mathcal{S}$ to the classical master equation $(\mathcal{S}, \mathcal{S}) = 0$ according to

$$s\Phi^*_A = (\mathcal{S}, \Phi^*_A) = \frac{\delta_R \mathcal{S}}{\delta \Phi^A}\,.$$

Owing to the off-shell closure of the gauge algebra $\mathcal{S}$ simply reads

$$\mathcal{S} = S_0 - \int (s\Phi^A)\Phi^*_A\,,$$

where S_0 is the classical action and $s\Phi^A$ are the BRST transformations given in section 2. It is useful to decompose the BRST differential according to the grading with respect to the antifield number $s = \sum_{k \geq -1} s_k$ with $\mathrm{agh}\,(s_k) = k$ (this decomposition should not be confused with the one in (4.3) even though we use the same notation). The decomposition starts with the field theoretical Koszul-Tate differential $\delta \equiv s_{-1}$ and the differential $\gamma \equiv s_0$. Contrary to the bosonic case the decomposition does not terminate at this level. An additional part s_1 raising the antifield number by one unit shows up reflecting field dependent gauge transformations in the commutator of supersymmetry transformations. The Koszul-Tate differential acts nontrivially only on the antifields and implements the equations of motion. Hence, the knowlegde of the classical action is necessary to determine the δ-transformations of the antifields. However, the action of the part of the BRST differential leaving the antifield number unchanged is determined solely by the imposed gauge transformations. The γ-transformations of the antifields corresponding to the matter fields and the U(1)

JHEP08(2001)023

multiplet read

$$
\begin{aligned}
\gamma X^*_M &= \partial_m(\xi^m X^*_M) - \mathrm{i}\partial_m(\xi^\beta(\gamma^m C)_{\beta\alpha}\psi^{*\alpha}_M) - \frac{1}{2}\partial_m(\xi^\alpha(\gamma^n\gamma^m C)_{\alpha\beta}\chi^\beta_n F^*_M) \\
\gamma\psi^{*\alpha}_M &= \partial_m(\xi^m\psi^{*\alpha}_M) + \xi^\alpha X^*_M - \mathrm{i}\xi^\gamma(\gamma^m C)_{\gamma\beta}\chi^\alpha_m\psi^{*\beta}_M - \frac{\mathrm{i}}{2}\partial_m(\xi^\beta(\gamma^m)_\beta{}^\alpha F^*_M) - \\
&\quad - \frac{\mathrm{i}}{8}\xi^\beta(\gamma^m\gamma_*)_\beta{}^\alpha\omega_m{}^{ab}\varepsilon_{ab}F^*_M - \frac{1}{2}\xi^\beta\chi_m{}^\delta(\gamma^m\gamma^n C)_{\beta\delta}\chi_n{}^\alpha F^*_M - \\
&\quad - \frac{1}{4}C^{ab}\varepsilon_{ab}(\gamma_*)_\beta{}^\alpha\psi^{*\beta}_M + \frac{1}{2}C^W\psi^{*\alpha}_M \\
\gamma F^*_M &= \partial_m(\xi^m F^*_M) - \xi^\beta C_{\beta\alpha}\psi^{*\alpha}_M - \frac{\mathrm{i}}{2}\xi^\beta(\gamma^m C)_{\beta\alpha}\chi^\alpha_m F^*_M + C^W F^*_M \\
\gamma A^{*m}_i &= \partial_n(\xi^n A^{*m}_i) - (\partial_n\xi^m)A^{*n}_i + \mathrm{i}\partial_n(\xi^\alpha(\gamma_* C)_{\alpha\beta}\varepsilon^{nm}\lambda^{*\beta}_i) \\
\gamma\phi^*_i &= \partial_m(\xi^m\phi^*_i) - \xi^\alpha(\gamma_* C)_{\alpha\beta}\varepsilon^{mn}\chi_n(\gamma_* C)\chi_m\lambda^{*\beta}_i - \mathrm{i}\partial_m(\xi^\alpha(\gamma_*\gamma^m C)_{\alpha\beta}\lambda^{*\beta}_i - \\
&\quad - \frac{\mathrm{i}}{2}\xi^\alpha(\gamma_* C)_{\alpha\beta}S\lambda^{*\beta}_i + 2\mathrm{i}\xi^\alpha\chi^\beta_m(\gamma_* C)_{\beta\alpha}A^{*m}_i - 2\eta^\gamma(\gamma_* C)_{\gamma\beta}\lambda^{*\beta}_i + C^W\phi^*_i \\
\gamma\lambda^{*\alpha}_i &= \partial_m(\xi^m\lambda^{*\alpha}_i) - \xi^\beta(\gamma_m)_\beta{}^\alpha A^{*m}_i + \xi^\beta(\gamma_*)_\beta{}^\alpha\phi^*_i - \mathrm{i}\xi^\beta(\gamma_*\gamma^m C)_{\beta\gamma}(\chi\gamma_*)^\alpha\lambda^{*\gamma}_i - \\
&\quad - \mathrm{i}\xi^\delta(\gamma_* C)_{\delta\beta}\varepsilon^{kl}\left(\chi^\gamma_k(\gamma_l)_\gamma{}^\alpha\right)\lambda^{*\beta}_i - \frac{1}{4}C^{ab}\varepsilon_{ab}(\gamma_*)_\beta{}^\alpha\lambda^{*\beta}_i + \frac{3}{2}C^W\lambda^{*\alpha}_i\,, \qquad (5.1)
\end{aligned}
$$

s_1 acts nontrivially on A^{*m}_i, ϕ^*_i and on the antifields for the gravitational multiplet χ^{*m}_α, e^{*m}_a and S^*. In particular one finds

$$
s_1 A^{*m}_i = \mathrm{i}\xi^\alpha\xi^\beta(\gamma^m C)_{\beta\alpha}c^*_i\,, \qquad s_1\phi^*_i = -\mathrm{i}\xi^\alpha\xi^\beta(\gamma_* C)_{\beta\alpha}c^*_i\,, \qquad (5.2)
$$

where c^*_i denote the antifields corresponding to U(1) ghosts.

The explicit form of the BRST transformations of the antifields for the gravitational multiplet and the ghosts will not be needed in the following. In section 7 it is shown that they do not contribute nontrivially to the cohomology, at least at ghost number $g < 4$.

5.1 Superconformal antifields

We shall now identify "superconformal antifields" whose γ-transformations take the same form as the s-transformations of superconformal tensor fields in (3.14). The identification of superconformal antifields is somewhat more involved than the procedure for the fields. From experience with the bosonic case one expects reasonable candidates to arise from redefinitions of the form $\Phi^*_A \to \frac{1}{1-\mu\bar\mu}\Phi^*_A$, accounting for the fact that antifields transform under diffeomorphisms as tensor densities rather than tensors. In addition we have to take care of their "structure group transformations", i.e., of their conformal weights, their Lorentz transformations and super-Weyl transformations.[10] Yet this does not suffice to obtain γ-transformations of the desired

[10] Antifields transform "contragradiently" under structure group transformations as compared to the corresponding fields.

form. It turns out that the antifields have to be mixed among themselves. These considerations lead us to the following definitions of the lowest order matter antifields

$$
\begin{aligned}
\hat{F}^*_M &\equiv F^*_{M(0,0)} = \frac{1}{1-\mu\bar{\mu}}(e_z{}^z e_{\bar{z}}{}^{\bar{z}})^{-\frac{1}{2}} F^*_M \\
\hat{\psi}^*_M &\equiv \psi^*_{M(0,0)} = \frac{\mathrm{i}}{\sqrt{2}}\frac{1}{1-\mu\bar{\mu}}(e_z{}^z)^{-\frac{1}{2}} \psi^*_M{}^2 + \frac{\mu\bar{\alpha}}{1-\mu\bar{\mu}}\hat{F}^*_M \\
\hat{\bar{\psi}}^*_M &\equiv \bar{\psi}^*_{M(0,0)} = \frac{\mathrm{i}}{\sqrt{2}}\frac{1}{1-\mu\bar{\mu}}(e_{\bar{z}}{}^{\bar{z}})^{-\frac{1}{2}} \psi^*_M{}^1 - \frac{\bar{\mu}\alpha}{1-\mu\bar{\mu}}\hat{F}^*_M \\
\hat{X}^*_M &\equiv X^*_{M(0,0)} = \frac{1}{1-\mu\bar{\mu}}X^*_M + \frac{\bar{\mu}\alpha}{1-\mu\bar{\mu}}\hat{\psi}^*_M + \frac{\mu\bar{\alpha}}{1-\mu\bar{\mu}}\hat{\bar{\psi}}^*_M + \frac{\alpha\bar{\alpha}}{1-\mu\bar{\mu}}\hat{F}^*_M \,.
\end{aligned}
$$

Their γ-transformations are indeed of the desired form (3.14) and read explicitly

$$
\begin{aligned}
\gamma\hat{F}^*_M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{F}^*_M - \varepsilon\hat{\bar{\psi}}^*_M + \bar{\varepsilon}\hat{\psi}^*_M + \frac{1}{2}((\partial\eta) + (\bar{\partial}\bar{\eta}))\hat{F}^*_M \\
\gamma\hat{\psi}^*_M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\psi}^*_M + \varepsilon\hat{X}^*_M + \bar{\varepsilon}\bar{\mathcal{D}}\hat{F}^*_M + (\frac{1}{2}(\partial\eta) + (\bar{\partial}\bar{\eta}))\hat{\psi}^*_M + (\bar{\partial}\bar{\varepsilon})\hat{F}^*_M \\
\gamma\hat{\bar{\psi}}^*_M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\bar{\psi}}^*_M + \bar{\varepsilon}\hat{X}^*_M - \varepsilon\mathcal{D}\hat{F}^*_M + ((\partial\eta) + \frac{1}{2}(\bar{\partial}\bar{\eta}))\hat{\bar{\psi}}^*_M - (\partial\varepsilon)\hat{F}^*_M \\
\gamma\hat{X}^*_M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{X}^*_M + \varepsilon\mathcal{D}\hat{\psi}^*_M + \bar{\varepsilon}\bar{\mathcal{D}}\hat{\bar{\psi}}^*_M + ((\partial\eta) + (\bar{\partial}\bar{\eta}))\hat{X}^*_M + (\partial\varepsilon)\hat{\psi}^*_M + \\
&\quad + (\bar{\partial}\bar{\varepsilon})\hat{\bar{\psi}}^*_M \,.
\end{aligned} \tag{5.3}
$$

The expressions above are in fact already complete, since s_1 does not act nontrivially on the matter antifields. Analogously to the situation of the superconformal tensor fields the algebra (3.12) is represented on these fields and their derivatives, which we denote by

$$
F^*_{M(m,n)} = (L_{-1})^m(\bar{L}_{-1})^n\hat{F}^*_M \equiv (\mathcal{D})^m(\bar{\mathcal{D}})^n\hat{F}^*_M \,,
$$

etc, where the operators L_{-1} and $\bar{L}_{-1}$ are identified with supercovariant derivatives as in (3.17). In particular one finds on the antifields with lowest conformal weights the following expressions

$$
\begin{aligned}
\mathcal{D}\hat{F}^*_M &= \frac{1}{1-\mu\bar{\mu}}\left((\partial - \bar{\mu}\bar{\partial} - \frac{1}{2}(\bar{\partial}\bar{\mu}) + \frac{1}{2}\bar{\mu}(\partial\mu))\hat{F}^*_M - \frac{1}{2}\bar{\mu}\alpha\hat{\bar{\psi}}^*_M - \frac{1}{2}\bar{\alpha}\hat{\psi}^*_M\right) \\
\bar{\mathcal{D}}\hat{F}^*_M &= \frac{1}{1-\mu\bar{\mu}}\left(\left(\bar{\partial} - \mu\partial - \frac{1}{2}(\partial\mu) + \frac{1}{2}\mu(\bar{\partial}\bar{\mu})\right)\hat{F}^*_M + \frac{1}{2}\alpha\hat{\bar{\psi}}^*_M + \frac{1}{2}\mu\bar{\alpha}\hat{\psi}^*_M\right) \\
\mathcal{D}\hat{\psi}^*_M &= \frac{1}{1-\mu\bar{\mu}}\left(\left(\partial - \bar{\mu}\bar{\partial} - (\bar{\partial}\bar{\mu}) + \frac{1}{2}\bar{\mu}(\partial\mu)\right)\hat{\psi}^*_M - \frac{1}{2}\bar{\mu}\alpha\hat{X}^*_M - \frac{1}{2}\bar{\alpha}\bar{\mathcal{D}}\hat{F}^*_M - \frac{1}{2}(\bar{\partial}\bar{\alpha})\hat{F}^*_M\right) \\
\bar{\mathcal{D}}\hat{\psi}^*_M &= \frac{1}{1-\mu\bar{\mu}}\left(\left(\bar{\partial} - \mu\partial - \frac{1}{2}(\partial\mu) + \mu(\bar{\partial}\bar{\mu})\right)\hat{\psi}^*_M + \frac{1}{2}\alpha\hat{X}^*_M + \frac{1}{2}\bar{\mu}\alpha\mathcal{D}\hat{F}^*_M + \frac{1}{2}\bar{\mu}(\partial\alpha)\hat{F}^*_M\right)
\end{aligned}
$$

and analogous formulas for $\mathcal{D}\hat{\bar{\psi}}^*_M$ and $\bar{\mathcal{D}}\hat{\bar{\psi}}^*_M$. Again higher order antifields will not be needed.

The construction of the covariant antifields for the gauge multiplet follows the arguments given above, with the additional task to get rid of the super-Weyl transformations. We introduce the redefinitions

$$
\begin{aligned}
\hat{\lambda}_i^* &\equiv \lambda^*_{i(0,0)} = -\frac{1}{1-\mu\bar{\mu}}(e_{\bar{z}}{}^{\bar{z}})^{-\frac{1}{2}}(e_z{}^z)^{-1}\lambda^{*2} \\
\hat{\bar{\lambda}}_i^* &\equiv \bar{\lambda}^*_{i(0,0)} = \frac{1}{1-\mu\bar{\mu}}(e_z{}^z)^{-\frac{1}{2}}(e_{\bar{z}}{}^{\bar{z}})^{-1}\lambda^{*1} \\
\hat{\phi}_i^* &\equiv \phi^*_{i(0,0)} = \frac{1}{\sqrt{2}}\frac{1}{1-\mu\bar{\mu}}(e_z{}^z)^{-\frac{1}{2}}(e_{\bar{z}}{}^{\bar{z}})^{-\frac{1}{2}}\phi_i^* - \frac{1}{2}\frac{1}{1-\mu\bar{\mu}}\left(\hat{\chi}_z{}^2 - \bar{\mu}\hat{\chi}_{\bar{z}}{}^2\right)\hat{\lambda}_i^* - \\
&\quad - \frac{1}{2}\frac{1}{1-\mu\bar{\mu}}\left(\hat{\chi}_{\bar{z}}{}^1 - \mu\hat{\chi}_z{}^1\right)\hat{\bar{\lambda}}_i^* \\
\hat{A}_i^* &\equiv A^*_{i(0,0)} = \frac{1}{\sqrt{2}}\frac{1}{1-\mu\bar{\mu}}\left(\bar{A}_i^* + \bar{\mu}A_i^*\right) - \frac{1}{1-\mu\bar{\mu}}\left(\bar{\alpha}\hat{\lambda}_i^* + \bar{\mu}\alpha\hat{\bar{\lambda}}_i^*\right) \\
\hat{\bar{A}}_i^* &\equiv \bar{A}^*_{i(0,0)} = \frac{1}{\sqrt{2}}\frac{1}{1-\mu\bar{\mu}}\left(A_i^* + \mu\bar{A}_i^*\right) - \frac{1}{1-\mu\bar{\mu}}\left(\alpha\hat{\bar{\lambda}}_i^* + \mu\bar{\alpha}\hat{\lambda}_i^*\right),
\end{aligned}
\tag{5.4}
$$

where we have used the shorthand notation for the corrections involving gravitions $\hat{\chi}_z{}^1 = \sqrt{\frac{8}{e_{\bar{z}}{}^{\bar{z}}}}\chi_z{}^1$ and $\hat{\chi}_z{}^2 = \sqrt{\frac{8}{e_z{}^z}}\chi_z{}^2$ with obvious expressions for the $\bar{z}$ components. The γ-transformations then read

$$
\begin{aligned}
\gamma\hat{\lambda}_i^* &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\lambda}_i^* + \frac{1}{2}\bar{\partial}\bar{\eta}\lambda_i^* + \varepsilon\hat{\phi}_i^* - \bar{\varepsilon}\hat{\bar{A}}_i^* \\
\gamma\hat{\bar{\lambda}}_i^* &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\bar{\lambda}}_i^* + \frac{1}{2}\partial\eta\hat{\bar{\lambda}}_i^* + \bar{\varepsilon}\hat{\phi}_i^* - \varepsilon\hat{A}_i^* \\
\gamma\hat{\phi}_i^* &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\phi}_i^* + \frac{1}{2}(\partial\eta + \bar{\partial}\bar{\eta})\hat{\phi}_i^* + \varepsilon\mathcal{D}\hat{\lambda}_i^* + \bar{\varepsilon}\bar{\mathcal{D}}\hat{\bar{\lambda}}_i^* \\
\gamma\hat{A}_i^* &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{A}_i^* + \partial\eta\hat{A}_i^* + \bar{\varepsilon}\mathcal{D}\hat{\lambda}_i^* - \varepsilon\mathcal{D}\hat{\bar{\lambda}}_i^* - \partial\varepsilon\hat{\bar{\lambda}}_i^* \\
\gamma\hat{\bar{A}}_i^* &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\hat{\bar{A}}_i^* + \bar{\partial}\bar{\eta}\hat{\bar{A}}_i^* + \varepsilon\bar{\mathcal{D}}\hat{\bar{\lambda}}_i^* - \bar{\varepsilon}\bar{\mathcal{D}}\hat{\lambda}_i^* - \bar{\partial}\bar{\varepsilon}\hat{\lambda}_i^*,
\end{aligned}
\tag{5.5}
$$

and are indeed of the desired form respecting the requirement (3.1). Note that the combination of the gravitinos used in the redefinition of $\hat{\phi}_i^*$ transforms into the super-Weyl ghost thereby removing the unwanted transformation properties under the super-Weyl symmetry. Again higher order antifields will not be needed.

The explicit form of the superconformal antifields given above has already been used to derive the results for the rigid symmetries presented in [6]. A complete list of the BRST transformations (including the Koszul-Tate part and the s_1-transformations) of the antifields needed for the cohomological analysis is given in appendix E. In the following sections (and also in the appendices) we have dropped the hats on the superconformal antifields, but it is clear from the context which set of variables is meant.

6. On-shell cohomology

We shall now define and analyse an "on-shell BRST cohomology" $H(\sigma)$ and show that it is isomorphic to its purely bosonic counterpart at ghost numbers < 4, i.e., to the on-shell BRST cohomology of the corresponding bosonic string model. The relevance of $H(\sigma)$ rests on the fact that it is isomorphic to the full local s-cohomology $H(s)$ (in the jet space associated to the fields and antifields), at least at ghost numbers < 4,

$$g < 4 : \quad H^g(\sigma) \simeq H^g(s) . \tag{6.1}$$

This will be proved in section 7.

The analysis in this and the next section is general, i.e., it applies to any model with an action (4.13) (or, equivalently, (4.14)) provided that two rather mild assumptions hold, which are introduced now. The first assumption only simplifies the action a little bit but does not reduce its generality: as we have argued already in [6], one may assume that the functions $D_i(X)$ which occur in the action coincide with a subset of the fields X^M. We denote this subset by $\{y^i\}$ and the remaining X's by x^μ,

$$\{X^M\} = \{x^\mu, y^i\}, \quad D_i(X) \equiv y^i . \tag{6.2}$$

For physical applications this "assumption" does not represent any loss of generality because it can always be achieved by a field redefinition ("target space coordinate transformation") $X^M \to \tilde{X}^M = \tilde{X}^M(X)$. The y^i may be interpreted as coordinates of an enlarged target space leading to "frozen extra dimensions" [6]. The second assumption is that $G_{\mu\nu}(x, y)$ is invertible (in contrast, G_{MN} need not be invertible). This is particularly natural in the string theory context, since it allows one to interpret $G_{\mu\nu}$ as a target space metric. It is rather likely that our result holds for even weaker assumptions (but we did not study this question), because the results derived in [19, 9] for bosonic string models do not use the invertibility of $G_{\mu\nu}$.

Let us remark that the isomorphism (6.1) is not too surprising, because it is reminiscent of a standard result of local BRST cohomology stating that $H(s)$ is isomorphic to the on-shell cohomology of γ in the space of antifield independent functions, where γ is the part of s with antifield number 0 (see, e.g., [10, section 7.2]). However, (6.1) is not quite the same statement because the definition of σ given below does not take the equations of motion for μ, $\bar{\mu}$, α or $\bar{\alpha}$ into account. Hence, (6.1) contains information in addition to the standard result of local BRST cohomology mentioned before: the equations of motion for μ, $\bar{\mu}$, α, $\bar{\alpha}$ are not relevant to the cohomology! This is a useful result as these equations of motion are somewhat unpleasant, because they are not linearizable (the models under study do not fulfill the standard regularity conditions described, e.g., in section 5.1 of [10]).

6.1 Definition of σ and $H(\sigma)$

σ is an "on-shell version" of s defined in the space of local functions made of the fields only (but not of any antifields). We work in the 'Beltrami basis' and use the equations of motion obtained by varying the action (4.13) with respect to the fields X, ψ, $\bar\psi$, $\hat F$, $\hat\phi$, λ, $\bar\lambda$ and A_m. The covariant version of these equations of motion can be obtained from the s-transformations of the corresponding covariant antifields given in appendix E by setting the antifield independent part ('Koszul-Tate part') of these transformations to zero. This gives the following "on-shell equalities" ($\approx$):

$$\hat F^i \approx 0 \tag{6.3}$$
$$\psi^i \approx 0 \tag{6.4}$$
$$\bar\psi^i \approx 0 \tag{6.5}$$
$$\mathcal{D}y^i \approx 0 \tag{6.6}$$
$$\bar{\mathcal{D}}y^i \approx 0 \tag{6.7}$$
$$\hat\phi^i \approx 2G_{i\mu}\hat F^\mu + \psi^\mu\bar\psi^\nu\Omega_{\mu\nu i} \tag{6.8}$$
$$\lambda^i \approx 2G_{i\mu}\mathcal{D}\bar\psi^\mu + \mathcal{D}x^\mu\bar\psi^\nu\Omega_{\mu\nu i} + \hat F^\mu\psi^\nu\Omega_{\nu i\mu} + \psi^\mu\psi^\nu\bar\psi^\rho R_{\nu\mu i\rho} \tag{6.9}$$
$$\bar\lambda^i \approx -2G_{i\mu}\bar{\mathcal{D}}\psi^\mu - \bar{\mathcal{D}}x^\mu\psi^\nu\Omega_{\nu\mu i} + \hat F^\mu\bar\psi^\nu\Omega_{i\nu\mu} + \psi^\mu\bar\psi^\nu\bar\psi^\rho R_{\mu i\rho\nu} \tag{6.10}$$
$$\hat F^\rho \approx -\frac{1}{2}\psi^\mu\bar\psi^\nu\Omega_{\mu\nu}{}^\rho \tag{6.11}$$
$$\bar{\mathcal{D}}\psi^\mu \approx -\frac{1}{2}[\bar{\mathcal{D}}x^\nu\psi^\rho\Omega_{\rho\nu}{}^\mu + \frac{1}{2}\psi^\lambda\bar\psi^\sigma\bar\psi^\rho\Omega_{\lambda\sigma}{}^\nu\Omega^\mu{}_{\rho\nu} + \psi^\nu\bar\psi^\rho\bar\psi^\sigma R^\mu{}_{\nu\sigma\rho}] \tag{6.12}$$
$$\mathcal{D}\bar\psi^\mu \approx \frac{1}{2}[-\mathcal{D}x^\nu\bar\psi^\rho\Omega_{\nu\rho}{}^\mu + \frac{1}{2}\psi^\lambda\bar\psi^\sigma\psi^\rho\Omega_{\lambda\sigma}{}^\nu\Omega_\rho{}^\mu{}_\nu + \psi^\nu\psi^\rho\bar\psi^\sigma R_{\rho\nu\sigma}{}^\mu] \tag{6.13}$$
$$\begin{aligned}\mathcal{F}^i \approx{}& 2G_{i\mu}\mathcal{D}\bar{\mathcal{D}}x^\mu + \mathcal{D}x^\mu\bar{\mathcal{D}}x^\nu\Omega_{\mu\nu i} - \hat F^\mu\hat F^\nu\Omega_{i\mu\nu} - \mathcal{D}\bar\psi^\mu\bar\psi^\nu\Omega_{i\nu\mu} + \psi^\mu\bar{\mathcal{D}}\psi^\nu\Omega_{\mu i\nu} - \\ &-\mathcal{D}x^\mu\bar\psi^\nu\bar\psi^\rho R_{\mu i\rho\nu} - \bar{\mathcal{D}}x^\mu\psi^\nu\psi^\rho R_{\rho\nu\mu i} - \hat F^\mu\psi^\nu\bar\psi^\rho\partial_i\Omega_{\nu\rho\mu} - \\ &-\frac{1}{2}\psi^\mu\psi^\nu\bar\psi^\rho\bar\psi^\sigma\partial_i R_{\nu\mu\sigma\rho}\end{aligned} \tag{6.14}$$
$$\begin{aligned}\mathcal{D}\bar{\mathcal{D}}x^\mu \approx{}& \frac{1}{2}\Big[-\mathcal{D}x^\nu\bar{\mathcal{D}}x^\rho\Omega_{\nu\rho}{}^\mu + \hat F^\nu\hat F^\rho\Omega^\mu{}_{\nu\rho} + \mathcal{D}\bar\psi^\nu\bar\psi^\rho\Omega^\mu{}_{\rho\nu} - \psi^\nu\bar{\mathcal{D}}\psi^\rho\Omega_\nu{}^\mu{}_\rho - \\ &- \mathcal{D}x^\sigma\bar\psi^\nu\bar\psi^\rho R^\mu{}_{\sigma\rho\nu} + \bar{\mathcal{D}}x^\sigma\psi^\nu\psi^\rho R_{\rho\nu\sigma}{}^\mu + \hat F^\sigma\psi^\nu\bar\psi^\rho\partial^\mu\Omega_{\nu\rho\sigma} + \\ &+ \frac{1}{2}\psi^\lambda\psi^\nu\bar\psi^\sigma\bar\psi^\rho\partial^\mu R_{\nu\lambda\rho\sigma}\Big],\end{aligned} \tag{6.15}$$

where indices μ of Ω, R, ∂ have been raised with the inverse of $G_{\mu\nu}(x,y)$, and ψ^i, $\bar\psi^i$ and $\hat F^i$ belong to the same supersymmetry multiplet as y^i (the auxiliary fields $\hat F^i$ should not be confused with the supercovariant field strengths $\mathcal{F}^i$ of the gauge fields). Note that the right hand sides of (6.8), (6.9), (6.10), (6.14) and (6.15) still contain $\hat F^\mu$, $\bar{\mathcal{D}}\psi^\mu$ or $\mathcal{D}\bar\psi^\mu$, which are to be substituted for by the expressions given in (6.11), (6.12) and (6.13), respectively. Furthermore, in (6.14) one has to substitute the expression resulting from (6.15) for $\mathcal{D}\bar{\mathcal{D}}x^\mu$. Using Eqs. (6.3) through (6.15) and their $\mathcal{D}$ and $\bar{\mathcal{D}}$ derivatives, we eliminate all variables on the left hand sides of these equations and all the covariant derivatives of these variables. Furthermore, we use these equations to define the σ-transformations of the remaining field variables from

JHEP08(2001)023

their s-transformations. For instance, one gets

$$\sigma y^i = 0 \tag{6.16}$$

$$\sigma x^\mu = (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})x^\mu + \varepsilon\psi^\mu + \bar\varepsilon\bar\psi^\mu \tag{6.17}$$

$$\begin{aligned}\sigma\psi^\mu &= \eta\mathcal{D}\psi^\mu - \frac{1}{2}\bar\eta\left[\bar{\mathcal{D}}x^\nu\psi^\rho\Omega_{\rho\nu}{}^\mu + \frac{1}{2}\psi^\lambda\bar\psi^\sigma\bar\psi^\rho\Omega_{\lambda\sigma}{}^\nu\Omega^\mu{}_{\rho\nu} + \psi^\nu\bar\psi^\rho\bar\psi^\sigma R^\mu{}_{\nu\sigma\rho}\right] + \\ &\quad + \frac{1}{2}\partial\eta\psi^\mu + \varepsilon\mathcal{D}x^\mu + \frac{1}{2}\bar\varepsilon\psi^\rho\bar\psi^\nu\Omega_{\rho\nu}{}^\mu\,. \end{aligned}\tag{6.18}$$

The σ-transformations of η, $\bar\eta$, ε, $\bar\varepsilon$, μ, $\bar\mu$, α, $\bar\alpha$ coincide with their s-transformations. The cohomology $H(\sigma)$ is the cohomology of σ in the space of local functions of the variables $\{u^\ell, v^\ell, W^A\}$, where the u's and v's are the same as in sections 3 and 4, while the W's are given by

$$\{W^A\} = \{y^i, x^\mu, \mathcal{D}^k x^\mu, \bar{\mathcal{D}}^k x^\mu, \mathcal{D}^r\psi^\mu, \bar{\mathcal{D}}^r\bar\psi^\mu, \partial^r\eta, \bar\partial^r\bar\eta, \partial^r\varepsilon, \bar\partial^r\bar\varepsilon, C^i : \\ k = 1, 2, \dots\ ,\ r = 0, 1, \dots\}\,. \tag{6.19}$$

$H(\sigma)$ is well-defined because σ squares to zero,

$$\sigma^2 = 0\,. \tag{6.20}$$

This holds because the (covariant) equations of motion of the fields X, ψ, $\bar\psi$, $\hat F$, $\hat\phi$, λ, $\bar\lambda$, A_m and their covariant derivatives transform into each other under diffeomorphisms and supersymmetry transformations but not into the equations of motion of μ, $\bar\mu$, α or $\bar\alpha$ [as can be read off from the s-transformations of the superconformal antifields in appendix E].

6.2 Relation to $H(\sigma, \mathcal{W})$

σ acts on the variables $\{u^\ell, v^\ell, W^A\}$ according to $\sigma u^\ell = v^\ell$, $\sigma W^A = r^A(W)$. Furthermore, analogously to (4.2) one has

$$\left\{\sigma\,,\,\frac{\partial}{\partial(\partial\eta)}\right\}W^A = L_0 W^A\,, \qquad \left\{\sigma\,,\,\frac{\partial}{\partial(\bar\partial\bar\eta)}\right\}W^A = \bar L_0 W^A\,, \tag{6.21}$$

i.e., in the space of local functions of the W's the derivatives with respect to $\partial\eta$ and $\bar\partial\bar\eta$ are contracting homotopies for L_0 and $\bar L_0$, respectively. Hence, the same standard arguments, which were used already in section 4 yield that $H(\sigma)$ is given by $H_{\mathrm{dR}}(GL^+(2)) \otimes H(\sigma, \mathcal{W})$, where $H_{\mathrm{dR}}(GL^+(2))$ reflects the nontrivial de Rham cohomology of the zweibein manifold (see [11, theorem 5.1]), while $H(\sigma, \mathcal{W})$ is the σ-cohomology in the space of local functions with vanishing conformal weights made solely of the variables (6.19),

$$H(\sigma) = H_{\mathrm{dR}}(\overset{+}{\mathrm{GL}}(2)) \otimes H(\sigma, \mathcal{W}), \quad \mathcal{W} = \{\omega : \omega = \omega(W),\ (L_0\omega, \bar L_0\omega) = (0,0)\}\,. \tag{6.22}$$

The factor $H_{\mathrm{dR}}(\mathrm{GL}^+(2))$ is irrelevant for the following discussion because it just reflects $\det e_m^a \neq 0$ and makes no difference between superstring and bosonic string models.

6.3 Decomposition of σ

To study $H(\sigma, \mathcal{W})$ we decompose σ into pieces of definite degree in the supersymmetry ghosts and the fermions.[11] The corresponding counting operator is denoted by N,

$$N = N_\varepsilon + N_{\bar\varepsilon} + N_\psi + N_{\bar\psi} \tag{6.23}$$

with N_ε and $N_{\bar\varepsilon}$ as in (4.4) and

$$N_\psi = \sum_{r\geq 0} (\mathcal{D}^r \psi^\mu) \frac{\partial}{\partial(\mathcal{D}^r \psi^\mu)} , \qquad N_{\bar\psi} = \sum_{r\geq 0} (\bar{\mathcal{D}}^r \bar\psi^\mu) \frac{\partial}{\partial(\bar{\mathcal{D}}^r \bar\psi^\mu)} .$$

Using the formulae given above, it is easy to verify that σ decomposes into pieces with even N-degree,

$$\sigma = \sum_{n\geq 0} \sigma_{2n} , \qquad [N, \sigma_{2n}] = 2n\, \sigma_{2n} , \tag{6.24}$$

where, on each variable (6.19), only finitely many σ_{2n} are non-vanishing. For instance, (6.18) yields

$$\begin{aligned}
\sigma_0 \psi^\mu &= \eta \mathcal{D}\psi^\mu - \frac{1}{2}\bar\eta \bar{\mathcal{D}} x^\nu \psi^\rho \Omega_{\rho\nu}{}^\mu + \frac{1}{2}\partial\eta\psi^\mu + \varepsilon \mathcal{D} x^\mu \\
\sigma_2 \psi^\mu &= -\frac{1}{4}\bar\eta \psi^\lambda \bar\psi^\sigma \bar\psi^\rho \Omega_{\lambda\sigma}{}^\nu \Omega^\mu{}_{\rho\nu} - \frac{1}{2}\bar\eta\psi^\nu\bar\psi^\rho\bar\psi^\sigma R^\mu{}_{\nu\sigma\rho} + \frac{1}{2}\bar\varepsilon\psi^\rho\bar\psi^\nu\Omega_{\rho\nu}{}^\mu \\
\sigma_{2n}\psi^\mu &= 0 \quad \text{for} \quad n > 1 .
\end{aligned}$$

6.4 Decomposition of σ_0

We shall prove the asserted result by an inspection of the cohomology of σ_0. To that end we decompose σ_0 according to the supersymmetry ghosts. That decomposition has only two pieces owing to the very definition of σ_0 and N,

$$\sigma_0 = \sigma_{0,0} + \sigma_{0,1} , \qquad [N_\varepsilon + N_{\bar\varepsilon}\, , \sigma_{0,0}] = 0 , \qquad [N_\varepsilon + N_{\bar\varepsilon}\, , \sigma_{0,1}] = \sigma_{0,1} . \tag{6.25}$$

[11] We are referring here to the variables (6.19) themselves, and not to the fermions that are implicitly contained in these variables through covariant derivatives.

One easily verifies by induction that $\sigma_{0,1}$ has the following simple structure:

$$
\begin{aligned}
\sigma_{0,1} y^i &= 0 \\
\sigma_{0,1} \mathcal{D}^r x^\mu &= 0 \\
\sigma_{0,1} \bar{\mathcal{D}}^r x^\mu &= 0 \\
\sigma_{0,1} \mathcal{D}^r \psi^\mu &= \sum_{k=0}^{r} \binom{r}{k} \partial^k \varepsilon\, \mathcal{D}^{r+1-k} x^\mu \\
\sigma_{0,1} \bar{\mathcal{D}}^r \bar{\psi}^\mu &= \sum_{k=0}^{r} \binom{r}{k} \bar{\partial}^k \bar{\varepsilon}\, \bar{\mathcal{D}}^{r+1-k} x^\mu \\
\sigma_{0,1} \partial^r \eta &= 0 \\
\sigma_{0,1} \bar{\partial}^r \bar{\eta} &= 0 \\
\sigma_{0,1} \partial^r \varepsilon &= 0 \\
\sigma_{0,1} \bar{\partial}^r \bar{\varepsilon} &= 0 \\
\sigma_{0,1} C^i &= 0\,.
\end{aligned}
\tag{6.26}
$$

6.5 $H(\sigma_0, \mathcal{W})$ at ghost numbers < 5

The cocycle condition of $H(\sigma_0, \mathcal{W})$ reads

$$
\sigma_0 \omega = 0, \quad \omega \in \mathcal{W}\,. \tag{6.27}
$$

We analyse (6.27) using (6.25). To that end we decompose ω according to the number of supersymmetry ghosts,

$$
\omega = \sum_{k=\underline{k}}^{\overline{k}} \omega_k\ , \quad (N_\varepsilon + N_{\bar{\varepsilon}})\omega_k = k\omega_k\,. \tag{6.28}
$$

Note that $\overline{k}$ is finite, $\overline{k} \leq \mathrm{gh}\,(\omega)$. Hence, the cocycle condition (6.27) decomposes into

$$
\sigma_{0,1}\omega_{\overline{k}} = 0\,, \qquad \sigma_{0,0}\omega_{\overline{k}} + \sigma_{0,1}\omega_{\overline{k}-1} = 0\,, \qquad \ldots \quad , \quad \sigma_{0,0}\omega_{\underline{k}} = 0\,. \tag{6.29}
$$

We can neglect contributions $\sigma_{0,1}\hat{\omega}_{\overline{k}-1}$ to $\omega_{\overline{k}}$ because such contributions can be removed by subtracting $\sigma_0\hat{\omega}_{\overline{k}-1}$ from ω. Hence, $\omega_{\overline{k}}$ can be assumed to be a nontrivial representative of $H(\sigma_{0,1}, \mathcal{W})$. That cohomology is computed in appendix A and yields

$$
\begin{aligned}
\omega_{\overline{k}} = {}& h(y, x, C, [\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}]) + \eta \mathcal{D} x^\mu h_\mu(y, x, \partial\eta, C, [\bar{\varepsilon}, \bar{\eta}]) + \\
& + \bar{\eta} \bar{\mathcal{D}} x^\mu \bar{h}_\mu(y, x, \bar{\partial}\bar{\eta}, C, [\varepsilon, \eta]) + \eta\bar{\eta} \mathcal{D} x^\mu \bar{\mathcal{D}} x^\nu h_{\mu\nu}(y, x, \partial\eta, \bar{\partial}\bar{\eta}, C)\,,
\end{aligned}
\tag{6.30}
$$

where $\sigma_{0,1}$-exact pieces have been neglected, and $[\varepsilon, \eta]$ and $[\bar{\varepsilon}, \bar{\eta}]$ denote dependence on the variables $\partial^r\varepsilon, \partial^r\eta$ and $\bar{\partial}^r\bar{\varepsilon}, \bar{\partial}^r\bar{\eta}$ $(r = 0, 1, \ldots)$, respectively. The result (6.30) holds for all ghost numbers and shows in particular that $\omega_{\overline{k}}$ can be assumed not to depend on the fermions $(\mathcal{D}^r\psi^\mu,\,\bar{\mathcal{D}}^r\bar\psi^\mu)$ at all. We now insert this result in the second equation (6.29), which requires that $\sigma_{0,0}\omega_{\overline{k}}$ be $\sigma_{0,1}$-exact. At ghost numbers < 5 this requirement kills completely the dependence of $\omega_{\overline{k}}$ on the supersymmetry ghosts as we show in appendix B. The result for these ghost numbers is thus that, modulo σ_0-exact pieces, the solutions to (6.27) neither depend on the fermions nor on the supersymmetry ghosts,

$$\begin{aligned}\mathrm{gh}\,(\omega) < 5:\ \ \omega = \sigma_0\hat\omega + h(y,x,C,[\eta],[\bar\eta]) + \eta\mathcal{D}x^\mu h_\mu(y,x,\partial\eta,C,[\bar\eta]) +\\ +\bar\eta\bar{\mathcal{D}}x^\mu\bar h_\mu(y,x,\bar\partial\bar\eta,C,[\eta]) + \eta\bar\eta\mathcal{D}x^\mu\bar{\mathcal{D}}x^\nu h_{\mu\nu}(y,x,\partial\eta,\bar\partial\bar\eta,C)\,.\end{aligned} \tag{6.31}$$

Furthermore, (6.25) and (6.26) show that a function which neither depends on the fermions nor on the supersymmetry ghosts is σ_0-exact if and only if it is the σ_0-transformation of a function which does not depend on these variables either. Combining this with (6.31) one concludes

$$g<5:\quad H^g(\sigma_0,\mathcal{W})\simeq H^g(\sigma_0,\mathcal{W}_0)\,, \tag{6.32}$$

where $\mathcal{W}_0$ is the subspace of $\mathcal{W}$ containing the functions with vanishing N-eigenvalues,

$$\mathcal{W}_0=\{\omega\in\mathcal{W}:N\omega=0\}\,.$$

This subspace can be made very explicit. The only variables (6.19) with negative conformal weights on which a function $\omega\in\mathcal{W}_0$ can depend are the undifferentiated ghosts η and $\bar\eta$ [note: the only other variables (6.19) with negative conformal weights are the undifferentiated supersymmetry ghosts, but they do not occur in $\omega\in\mathcal{W}_0$ by the very definition of $\mathcal{W}_0$]. Since η and $\bar\eta$ are anticommuting variables each of them can occur at most once in a monomial contributing to $\omega\in\mathcal{W}_0$. Hence, since η and $\bar\eta$ have conformal weights $(-1,0)$ and $(0,-1)$, respectively, functions in $\mathcal{W}_0$ can only depend on those w's with conformal weights ≤ 1 (as higher weights cannot be compensated for by variables with negative weights), and a variable with L_0-weight ($\bar L_0$-weight) 1 must necessarily occur together with η ($\bar\eta$). This yields

$$\omega\in\mathcal{W}_0\ \Leftrightarrow\ \omega=f(y,x,C,\partial\eta,\bar\partial\bar\eta,\eta\mathcal{D}x^\mu,\bar\eta\bar{\mathcal{D}}x^\mu,\eta\partial^2\eta,\bar\eta\bar\partial^2\bar\eta). \tag{6.33}$$

Note that $H(\sigma_0,\mathcal{W}_0)$ is nothing but the on-shell cohomology $H(\sigma,\mathcal{W})$ of the corresponding bosonic string model, since elements of $\mathcal{W}_0$ neither depend on the fermions nor on the supersymmetry ghosts, and since σ_0 reduces in $\mathcal{W}_0$ to $\sigma_{0,0}$, which encodes only the diffeomorphism transformations but not the supersymmetry transformations.

6.6 $H(\sigma)$ at ghost numbers < 4

We shall now show that $H(\sigma,\mathcal{W})$ is at ghost numbers < 4 isomorphic to $H(\sigma_0,\mathcal{W}_0)$,

$$g<4:\quad H^g(\sigma,\mathcal{W})\simeq H^g(\sigma_0,\mathcal{W}_0)\,. \tag{6.34}$$

Because of (6.22) this implies that $H(\sigma)$ is isomorphic to its counterpart in the corresponding bosonic string model (recall that the factor $H_{\mathrm{dR}}(GL^+(2))$ is present in the case of bosonic strings as well, and that $H^g(\sigma_0,\mathcal{W}_0)$ is the on-shell cohomology of the bosonic string model). To derive (6.34), we consider the cocycle condition of $H(\sigma,\mathcal{W})$,

$$\sigma\omega = 0, \quad \omega \in \mathcal{W}\,. \tag{6.35}$$

We decompose ω into pieces with definite degree in the supersymmetry ghosts and fermions,

$$\omega = \sum_{n=\underline{n}}^{\overline{n}} \omega_n, \quad N\omega_n = n\,\omega_n\,, \tag{6.36}$$

with N as in (6.23) [actually there are only even values of n in this decomposition because ω has vanishing conformal weights]. The cocycle condition (6.35) implies in particular

$$\sigma_0\omega_{\underline{n}} = 0\,, \tag{6.37}$$

where we used the decomposition (6.24) of σ. Hence, every cocycle ω of $H^g(\sigma,\mathcal{W})$ contains a coycle $\omega_{\underline{n}}$ of $H^g(\sigma_0,\mathcal{W})$. Our result (6.32) on $H^g(\sigma_0,\mathcal{W})$ implies that this relation between representatives of $H^g(\sigma,\mathcal{W})$ and $H^g(\sigma_0,\mathcal{W})$ gives rise to a one-to-one correspondence between the cohomology classes of $H^g(\sigma,\mathcal{W})$ and $H^g(\sigma_0,\mathcal{W}_0)$ for $g<4$ and thus to (6.34). The arguments are standard and essentially the following:

(i) When $g<5$, $\omega_{\underline{n}}$ can be assumed to be nontrivial in $H^g(\sigma_0,\mathcal{W})$ and represents thus a class of $H^g(\sigma_0,\mathcal{W}_0)$. Indeed, assume it were trivial, i.e., $\omega_{\underline{n}} = \sigma_0\hat\omega_{\underline{n}}$ for some $\hat\omega_{\underline{n}}\in\mathcal{W}$. In that case we can remove $\omega_{\underline{n}}$ from ω by subtracting $\sigma\hat\omega_{\underline{n}}$. $\omega' := \omega-\sigma\hat\omega_{\underline{n}}\in\mathcal{W}$ is cohomologically equivalent to ω and its decomposition (6.36) starts at some degree $\underline{n}'>\underline{n}$ unless it vanishes (which implies already $\omega=\sigma\hat\omega_{\underline{n}}$). The cocycle condition for ω' implies $\sigma_0\omega'_{\underline{n}'}=0$ and thus $\omega'_{\underline{n}'}=\sigma_0\hat\omega'_{\underline{n}'}$ for some $\hat\omega'_{\underline{n}'}\in\mathcal{W}$ as a consequence of (6.32) (owing to $\underline{n}'>\underline{n}\geq 0$). Repeating the arguments, one concludes that ω is σ-exact, $\omega=\sigma(\hat\omega_{\underline{n}}+\hat\omega'_{\underline{n}'}+\dots)$ [it is guaranteed that the procedure terminates, i.e., that the sum $\hat\omega_{\underline{n}}+\hat\omega'_{\underline{n}'}+\dots$ is finite and thus local, because the number of supersymmetry ghosts is bounded by the ghost number and thus the number of fermions is bounded too because ω has vanishing conformal weights].

(ii) When $g<4$, every nontrivial cocycle ω_0 of $H^g(\sigma_0,\mathcal{W}_0)$ can be completed to a nontrivial cocycle ω of $H^g(\sigma,\mathcal{W})$. Indeed suppose we had constructed $\omega_n\in\mathcal{W}$, $n=0,\dots,m$ with ghost number g such that $\omega^{(m)}:=\sum_{n=0}^m\omega_n$ fulfills $\sigma\omega^{(m)}=\sum_{n\geq m+1}R_n$ with $NR_n=nR_n$ [for $m=0$ this is implied by $\sigma_0\omega_0=0$ which holds because ω_0 is a σ_0-cocycle by assumption]. $\sigma^2=0$ implies $\sigma\sum_{n\geq m+1}R_n=0$ and thus $\sigma_0R_{m+1}=0$ at lowest N-degree. Note that R_{m+1} is in $\mathcal{W}$ (owing to $\sigma\mathcal{W}\subset\mathcal{W}$) and that it has ghost number $g+1<5$ because $\omega^{(m)}$ has ghost number $g<4$. (6.32) guarantees thus that there is some $\omega_{m+1}\in\mathcal{W}$ such that $R_{m+1}=-\sigma_0\omega_{m+1}$, which implies that $\omega^{(m+1)}:=\omega^{(m)}+\omega_{m+1}$ fulfills $\sigma\omega^{(m+1)}=\sum_{n\geq m+2}R'_n$. By induction this implies that every solution to (6.37) with ghost number <4 can indeed be completed to a solution of (6.35) [the locality of ω holds by the same arguments as above]. If ω_0 is trivial in $H^g(\sigma_0,\mathcal{W}_0)$, then its completion ω is trivial in $H^g(\sigma,\mathcal{W})$ by arguments used in (i). Conversely, the triviality of ω in $H^g(\sigma,\mathcal{W})$ ($\omega=\sigma\eta$) implies obviously the triviality of ω_0 in $H^g(\sigma_0,\mathcal{W}_0)$ ($\omega_0=\sigma_0\eta_0$) because there are no negative N-degrees.

7. Relation to the cohomology of bosonic strings

We shall now derive (6.1) and the announced isomorphism between the s-cohomologies of a superstring and the corresponding bosonic string model. Both results can be traced to the existence of variables $\{\tilde u^{\tilde\ell},\tilde v^{\tilde\ell},\tilde W^{\tilde A}\}$ on which s takes a form very similar to σ on the variables $\{u^\ell,v^\ell,w^A\}$ used in section 6. In the 'Beltrami basis' the set of $\tilde u$'s consists of: (i) $\tilde u$'s with ghost number 0 which coincide with the u^ℓ; (ii) $\tilde u$'s with ghost number -1 given by the superconformal antifields X^*_M, ψ^*_M, $\bar\psi^*_M$, F^*_M, ϕ^*_i, λ^*_i $\bar\lambda^*_i$, A^*_i (recall that we have dropped the hats on these antifields) and all covariant derivatives of these antifields plus the $\bar A^*_i$ and all their $\bar{\mathcal{D}}$-derivatives ($\bar{\mathcal{D}}^r\bar A^*_i$, $r=0,1,\dots$);[12] (iii) $\tilde u$'s with ghost number -2 given by the antifields of the ghosts, i.e., by η^*, $\bar\eta^*$, ε^*, $\bar\varepsilon^*$, C^*_i and all their derivatives. It can be readily checked that a complete set of new local jet coordinates in the Beltrami basis is given by $\{\tilde u^{\tilde\ell},\tilde v^{\tilde\ell},\tilde W^{\tilde A}_{(0)}\}$ with $\tilde v^{\tilde\ell}=s\tilde u^{\tilde\ell}$ and

$$\{\tilde W^{\tilde A}_{(0)}\}=\{y^i,x^\mu,\mathcal{D}^kx^\mu,\bar{\mathcal{D}}^kx^\mu,\mathcal{D}^r\psi^\mu,\bar{\mathcal{D}}^r\bar\psi^\mu,\partial^r\eta,\bar\partial^r\bar\eta,\partial^r\varepsilon,\bar\partial^r\bar\varepsilon,C^i,\\ \partial^r\mu^*,\bar\partial^r\bar\mu^*,\partial^r\alpha^*,\bar\partial^r\bar\alpha^*:k=1,2,\dots\ ,\ r=0,1,\dots\}. \tag{7.1}$$

Note that $\{\tilde W^{\tilde A}_{(0)}\}$ does not only contain the W^A listed in (6.19), but in addition the variables $\partial^r\mu^*$, $\bar\partial^r\bar\mu^*$, $\partial^r\alpha^*$, $\bar\partial^r\bar\alpha^*$. The latter occur here because their s-transformations contain no linear parts and can therefore not be used as $\tilde v$'s.[13] The $\tilde W^{\tilde A}_{(0)}$ fulfull

$$s\tilde W^{\tilde A}_{(0)}=r^{\tilde A}(\tilde W_{(0)})+O(1)\,, \tag{7.2}$$

[12]The $\mathcal{D}^k\bar{\mathcal{D}}^r\bar A^*_i$ with $k>0$ do not count among the u's because the antifield independent parts of $s\mathcal{D}^k\bar{\mathcal{D}}^r\bar A^*_i$ and $-s\mathcal{D}^{k-1}\bar{\mathcal{D}}^{r+1}A^*_i$ are equal (both are given by $\mathcal{D}^k\bar{\mathcal{D}}^{r+1}y^i$). Rather, they are substituted for by the v's corresponding to the $\mathcal{D}^{k-1}\bar{\mathcal{D}}^rC^*_i$ ($k>0$) owing to $s\mathcal{D}^{k-1}\bar{\mathcal{D}}^rC^*_i=-\mathcal{D}^k\bar{\mathcal{D}}^r\bar A^*_i+\dots$.

[13]The other derivatives of the antifields μ^*, $\bar\mu^*$, α^*, $\bar\alpha^*$, such as the $\bar\partial^k\partial^r\mu^*$ ($k>0$), do not occur among the $\tilde W_{(0)}$'s because they are substituted for by the $\tilde v$'s corresponding to η^*, $\bar\eta^*$, ε^*, $\bar\varepsilon^*$ and their derivatives (e.g., one has $s\eta^*=-\bar\partial\mu^*+\dots$).

where $O(1)$ collects terms which are at least linear in the $\tilde{u}$'s and $\tilde{v}$'s. As shown in [27], (7.2) implies the existence of variables $\tilde{W}^{\tilde{A}} = \tilde{W}^{\tilde{A}}_{(0)} + O(1)$ which fulfill

$$s\tilde{W}^{\tilde{A}} = r^{\tilde{A}}(\tilde{W}) \tag{7.3}$$

with the *same* functions $r^{\tilde{A}}$ as in (7.2). Furthermore the algorithm described in [27] for the construction of the $\tilde{W}^{\tilde{A}}$ results in local expressions when applied in the present case. This can be shown by means of arguments similar to those used within the discussion of the examples in [27].[14]

(7.3) implies that the s-transformations of those $\tilde{W}$'s which correspond to the variables (6.19) can be obtained from the σ-transformations of the latter variables simply by substituting there $\tilde{W}$'s for the corresponding W's. For instance, this gives

$$sy^{i\prime} = 0\,, \tag{7.4}$$
$$sx^{\mu\prime} = \eta(\mathcal{D}x^\mu)' + \bar{\eta}(\bar{\mathcal{D}}x^\mu)' + \varepsilon\psi^{\mu\prime} + \bar{\varepsilon}\bar{\psi}^{\mu\prime}\,, \tag{7.5}$$

where here and in the following a prime on a variable indicates a $\tilde{W}$-variable.[15] For instance, $y^{i\prime}$ is the $\tilde{W}$-variable corresponding to y^i and explicitly given by

$$y^{i\prime} = y^i + \varepsilon\bar{\lambda}^*_i - \bar{\varepsilon}\lambda^*_i - \eta A^*_i + \bar{\eta}\bar{A}^*_i + \eta\bar{\eta}C^*_i\,. \tag{7.6}$$

This very close relation between s on the $\tilde{W}$-variables and σ on the variables (6.19) would immediately imply $H(s) \simeq H(\sigma)$ if the $\tilde{W}$-variables $(\partial^r\mu^*)'$, $(\bar{\partial}^r\bar{\mu}^*)'$, $(\partial^r\alpha^*)'$, $(\bar{\partial}^r\bar{\alpha}^*)'$ were not present. Nevertheless the asserted isomorphism (6.1) holds because the conformal weights of the latter variables are too high so that they cannot contribute nontrivially to $H^g(s)$ for $g < 4$. To show this we analyse $H(s)$ along the same lines as $H(\sigma)$ in section 6.

The first step of that analysis gives

$$H(s) \simeq H_{\mathrm{dR}}(GL^+(2)) \otimes H(s, \tilde{\mathcal{W}}), \quad \tilde{\mathcal{W}} = \{\omega : \omega = \omega(w),\ (L_0\omega, \bar{L}_0\omega) = (0,0)\}\,. \tag{7.7}$$

This result is analogous to (6.22) and expresses that the zweibein gives the only nontrivial cohomology in the subspace of $\tilde{u}$'s and $\tilde{v}$'s and that there is a contracting homotopy for L_0 and $\bar{L}_0$ because (7.3) implies

$$\left\{s\,, \frac{\partial}{\partial(\partial\eta)}\right\}\tilde{W}^{\tilde{A}} = L_0\tilde{W}^{\tilde{A}}\ , \quad \left\{s\,, \frac{\partial}{\partial(\bar{\partial}\bar{\eta})}\right\}\tilde{W}^{\tilde{A}} = \bar{L}_0\tilde{W}^{\tilde{A}}\,.$$

[14] In the present case, the suitable 'degrees' to be used in these arguments are the conformal weights and the ghost number. Using these degrees one can prove that the algorithm produces local (though not necessarily polynomial) expressions: the resulting $\tilde{W}$'s can depend nonpolynomially on the x^μ, y^i and on the two particular combinations $\varepsilon\bar{\lambda}^*_i$ and $\bar{\varepsilon}\lambda^*_i$ but they are necessarily polynomials in all variables which contain derivatives of fields or antifields.

[15] The construction of the $\tilde{W}$'s implies $(\partial^r\eta)' = \partial^r\eta$, $(\bar{\partial}^r\bar{\eta})' = \bar{\partial}^r\bar{\eta}$, $(\partial^r\varepsilon)' = \partial^r\varepsilon$ and $(\bar{\partial}^r\bar{\varepsilon})' = \bar{\partial}^r\bar{\varepsilon}$ because the s-transformation of these ghost variables do not contain any $\tilde{u}$'s or $\tilde{v}$'s. This has been used in (7.5).

JHEP08(2001)023

The conformal weights of $\alpha^{*\prime}$, $\bar{\alpha}^{*\prime}$, $\mu^{*\prime}$ and $\bar{\mu}^{*\prime}$ are $(3/2,0)$, $(0,3/2)$, $(2,0)$ and $(0,2)$, respectively.

$H(s,\tilde{\mathcal{W}})$ can be analysed by means of a decomposition of s analogous to the σ-decomposition in (6.24), using a counting operator N' for all those $\tilde{W}$'s which have half-integer conformal weights,

$$N' = N_{\varepsilon} + N_{\bar{\varepsilon}} + N_{\psi'} + N_{\bar{\psi}'} + N_{\alpha^{*\prime}} + N_{\bar{\alpha}^{*\prime}} \, .$$

The decomposition of s reads

$$s = \sum_{n\geq 0} s_{2n} \quad , \quad [N', s_{2n}] = 2n\, s_{2n} \, .$$

Next we examine the s_0-cohomology. Analogously to (6.25) one has

$$s_0 = s_{0,0} + s_{0,1} \, , \qquad [N_{\varepsilon} + N_{\bar{\varepsilon}} \, , s_{0,0}] = 0 \, , \qquad [N_{\varepsilon} + N_{\bar{\varepsilon}} \, , s_{0,1}] = s_{0,1} \, .$$

We now determine the cohomology of $s_{0,1}$ along the lines of the investigation of the $\sigma_{0,1}$-cohomology in appendix A by inspecting the part of $s_{0,1}$ which contains the undifferentiated ghost ε. That part is the analog of $\sigma_{0,1,1}$ in (A.2) and takes the form $\varepsilon\, \hat{G}'_{-1/2}$. $\hat{G}'_{-1/2}$ acts nontrivially only on the ψ', $\alpha^{*\prime}$ and their (covariant) derivatives according to

$$\hat{G}'_{-1/2}(\mathcal{D}^r\psi^{\mu})' = (\mathcal{D}^{r+1}x^{\mu})' \quad , \quad \hat{G}'_{-1/2}(\partial^r\alpha^*)' = -(\partial^r\mu^*)' \, .$$

We define a contracting homotopy B' which is analogous to the contracting homotopy B in appendix A,

$$B' = \sum_{r\geq 0}\left[(\mathcal{D}^r\psi^{\mu})' \, \frac{\partial}{\partial(\mathcal{D}^{r+1}x^{\mu})'} - (\partial^r\alpha^*)' \, \frac{\partial}{\partial(\partial^r\mu^*)'}\right] .$$

Using B' one proves that the functions f'_m with $m > 0$ which are analogous to the functions f_m in appendix A can be assumed not to depend on the variables $(\mathcal{D}^r\psi^{\mu})'$, $(\mathcal{D}^{r+1}x^{\mu})'$, $(\partial^r\alpha^*)'$ or $(\partial^r\mu^*)'$.[16] In the case $m = 0$ one gets that f'_0 does not depend on $(\partial^r\alpha^*)'$ or $(\partial^r\mu^*)'$, simply because the conformal weights of these variables are too large [cf. the arguments in the text after (A.9)]. This implies the analog of equation (A.11), with functions h'_m and g'_{μ} which may still depend on $(\bar{\mathcal{D}}^r\bar{\psi}^{\mu})'$, $(\bar{\mathcal{D}}^{r+1}x^{\mu})'$, $(\bar{\partial}^r\bar{\alpha}^*)'$ or $(\bar{\partial}^r\bar{\mu}^*)'$. The dependence on these variables can be

[16]For this argument it is important that there is a finite maximal value $\overline{m}$ of m. In the case of the σ-cohomology, m was bounded from above by the ghost number but now the ghost number alone does not give a bound because there are variables with negative ghost numbers, the $(\partial^r\alpha^*)'$, $(\bar{\partial}^r\bar{\alpha}^*)'$, $(\partial^r\mu^*)'$ and $(\bar{\partial}^r\bar{\mu}^*)'$. Nevertheless there is a bound because $\omega(\tilde{W})$ does not only have fixed ghost number but also vanishing conformal weights. Indeed, it is easy to show that this forbids arbitrarily large powers of ε because the $(\partial^r\alpha^*)'$ and $(\partial^r\mu^*)'$ have ghost number -1 and conformal weights $\geq 3/2$.

analysed analogously, using a contracting homotopy $\bar{B}'$ for these variables, along the lines of the remaining analysis in appendix A. One finally obtains the following result for $H(s_{0,1},\tilde{\mathcal{W}})$:

$$\begin{aligned} s_{0,1}\omega = 0\,, \qquad & \omega\in\tilde{\mathcal{W}} \quad \Rightarrow \\ \omega = {} & h(y',x',C',[\varepsilon,\eta],[\bar{\varepsilon},\bar{\eta}]) + \\ & + \eta(\mathcal{D}x^\mu)'h_\mu(y',x',\partial\eta,C',[\bar{\varepsilon},\bar{\eta}]) + \bar{\eta}(\bar{\mathcal{D}}x^\mu)'\bar{h}_\mu(y',x',\bar{\partial}\bar{\eta},C',[\varepsilon,\eta]) + \\ & + \eta\bar{\eta}(\mathcal{D}x^\mu)'(\bar{\mathcal{D}}x^\nu)'h_{\mu\nu}(y',x',\partial\eta,\bar{\partial}\bar{\eta},C') + s_{0,1}\hat{\omega}(w),\ \hat{\omega}\in\tilde{\mathcal{W}}\,. \end{aligned} \tag{7.8}$$

Hence, $H(s_{0,1},\tilde{\mathcal{W}})$ is completely isomorphic to $H(\sigma_{0,1},\mathcal{W})$ (for all ghost numbers). In particular, the representatives do not depend on $(\partial^r\alpha^*)'$, $(\bar{\partial}^r\bar{\alpha}^*)'$, $(\partial^r\mu^*)'$ or $(\bar{\partial}^r\bar{\mu}^*)'$ [recall that the reason is that the conformal weights of these variables are too high; if, for instance, $\mu^{*\prime}$ had conformal weights $(1,0)$ instead of $(2,0)$ it had contributed to (7.8) analogously to $(\mathcal{D}x^\mu)'$]. This implies the results announced above: arguments which are completely analogous to those used to derive first (6.31) and then (6.34) lead to

$$g<4: \quad H^g(s,\tilde{\mathcal{W}})\simeq H^g(s_0,\tilde{\mathcal{W}}_0), \quad \tilde{\mathcal{W}}_0=\{\omega\in\tilde{\mathcal{W}}: N'\omega=0\}\,. \tag{7.9}$$

Analogously to (6.33), the elements of $\tilde{\mathcal{W}}_0$ can only depend on those w's with conformal weights ≤ 1, i.e.,

$$\omega'\in\tilde{\mathcal{W}}_0 \ \Leftrightarrow\ \omega' = f(y',x',C',\partial\eta,\bar{\partial}\bar{\eta},\eta(\mathcal{D}x^\mu)',\bar{\eta}(\bar{\mathcal{D}}x^\mu)',\eta\partial^2\eta,\bar{\eta}\bar{\partial}^2\bar{\eta})\,. \tag{7.10}$$

Because of (7.3), s_0 takes exactly the same form in $\tilde{\mathcal{W}}_0$ as σ_0 in $\mathcal{W}_0$. This implies (for all ghost numbers)

$$H(s_0,\tilde{\mathcal{W}}_0)\simeq H(\sigma_0,\mathcal{W}_0)\,. \tag{7.11}$$

Because of (7.9) and (6.34) (as well as (7.7) and (6.22)) this yields (6.1). (7.9) establishes also the equivalence between the cohomologies of the superstring and the corresponding bosonic string at ghost numbers <4 because $H_{\mathrm{dR}}(GL^+(2))\otimes H(s_0,\tilde{\mathcal{W}}_0)$ is nothing but the s-cohomology of the bosonic string.

Acknowledgments

A.K. was supported by the ÖNB under grant number 7731 and by the Austrian Research Fund FWF under grant number P14639-TPH.

A. Cohomology of $\sigma_{0,1}$ in $\mathcal{W}$

In this appendix we compute $H(\sigma_{0,1},\mathcal{W})$ where $\sigma_{0,1}$ is given in (6.26). The cocycle condition reads

$$\sigma_{0,1}\omega=0\,, \qquad \omega\in\mathcal{W}\,. \tag{A.1}$$

We decompose this equation into pieces with definite degree in the undifferentiated supersymmetry ghosts ε. $\sigma_{0,1}$ decomposes into two pieces, $\sigma_{0,1,0}$ and $\sigma_{0,1,1}$, where $\sigma_{0,1,0}$ does not change the degree in the undifferentiated ε, whereas $\sigma_{0,1,1}$ increases this degree by one unit. $\sigma_{0,1,1}$ reads

$$\sigma_{0,1,1} = \varepsilon\, \hat{G}_{-1/2}\,, \qquad \hat{G}_{-1/2} = \sum_{r\geq 0}(\mathcal{D}^{r+1}x^\mu)\,\frac{\partial}{\partial(\mathcal{D}^r\psi^\mu)}\,. \tag{A.2}$$

ω can be assumed to have fixed ghost number and is thus a polynomial in the undifferentiated ε,

$$\omega = \sum_{m=\underline{m}}^{\overline{m}} \varepsilon^m f_m\,, \tag{A.3}$$

where f_m can depend on all variables (6.19) except for the undifferentiated ε. At highest degree in the undifferentiated ε, (A.1) implies $\sigma_{0,1,1}(\varepsilon^{\overline{m}} f_{\overline{m}}) = 0$ and thus

$$\hat{G}_{-1/2} f_{\overline{m}} = 0\,. \tag{A.4}$$

We analyse this condition by means of the contracting homotopy

$$B = \sum_{r\geq 0}(\mathcal{D}^r\psi^\mu)\,\frac{\partial}{\partial(\mathcal{D}^{r+1}x^\mu)}\,.$$

The anticommutator of B and $\hat{G}_{-1/2}$ is the counting operator for all variables $\mathcal{D}^r\psi^\mu$ and $\mathcal{D}^{r+1}x^\mu$ $(r = 0, 1, \dots)$,

$$\{B, \hat{G}_{-1/2}\} = \sum_{r\geq 0}\left[(\mathcal{D}^r\psi^\mu)\,\frac{\partial}{\partial(\mathcal{D}^r\psi^\mu)} + (\mathcal{D}^{r+1}x^\mu)\,\frac{\partial}{\partial(\mathcal{D}^{r+1}x^\mu)}\right].$$

Hence, (A.4) implies by standard arguments that $f_{\overline{m}}$ is $\hat{G}_{-1/2}$-exact up to a function that does not depend on the $\mathcal{D}^r\psi^\mu$ or $\mathcal{D}^{r+1}x^\mu$,

$$f_{\overline{m}} = \hat{G}_{-1/2}\, g_{\overline{m}} + h_{\overline{m}}(y, x, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\partial\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}])\,, \tag{A.5}$$

where $g_{\overline{m}}$ is a function that can depend on all variables (6.19) except for the undifferentiated ε, $[\bar{\mathcal{D}}x, \bar{\psi}]$ denotes collectively the variables $\bar{\mathcal{D}}^{r+1}x^\mu, \bar{\mathcal{D}}^r\bar{\psi}^\mu$, and $[\partial\varepsilon, \eta]$ and $[\bar{\varepsilon}, \bar{\eta}]$ denote collectively the variables $\partial^{r+1}\varepsilon, \partial^r\eta$ and $\bar{\partial}^r\bar{\varepsilon}, \bar{\partial}^r\bar{\eta}$, respectively ($r = 0, 1, \dots$ in all cases). We shall first study the case $\overline{m} > 0$ [the case $\overline{m} = 0$ will be included automatically below]. (A.5) implies

$$\begin{aligned}\overline{m} > 0: \quad \omega = {} & \sigma_{0,1}(\varepsilon^{\overline{m}-1} g_{\overline{m}}) + \varepsilon^{\overline{m}-1} f'_{\overline{m}-1} + \sum_{m=\underline{m}}^{\overline{m}-2} \varepsilon^m f_m + \\ & + \varepsilon^{\overline{m}} h_{\overline{m}}(y, x, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\partial\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}])\,,\end{aligned} \tag{A.6}$$

where

$$f'_{\overline{m}-1} = f_{\overline{m}-1} - \sigma_{0,1,0}\, g_{\overline{m}}\,.$$

The exact piece $\sigma_{0,1}(\varepsilon^{\overline{m}-1}g_{\overline{m}})$ on the right hand side of (A.6) will be neglected in the following, i.e., actually we shall examine $\omega' := \omega - \sigma_{0,1}(\varepsilon^{\overline{m}-1}g_{\overline{m}})$ in the following. However, for notational convenience, we shall drop the primes (of ω' and $f'_{\overline{m}-1}$) and consider now

$$\overline{m} > 0: \quad \omega = \sum_{m=\underline{m}}^{\overline{m}-1} \varepsilon^m f_m + \varepsilon^{\overline{m}} h_{\overline{m}}(y, x, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\partial\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}])\,. \tag{A.7}$$

We have thus learned that, if $\overline{m} > 0$, the piece in ω with highest degree in the undifferentiated ε can be assumed not to depend on any of the variables $\mathcal{D}^r\psi^\mu$ or $\mathcal{D}^{r+1}x^\mu$ $(r = 0, 1, \ldots)$. As a consquence, the $\sigma_{0,1}$-transformation of that piece does not depend on these variables either and $\sigma_{0,1}\omega = 0$, with ω as in (A.7), implies

$$\hat{G}_{-1/2} f_{\overline{m}-1} = 0\,. \tag{A.8}$$

We can now analyse (A.8) in the same way as (A.4) and repeat the arguments until we reach an equation

$$\hat{G}_{-1/2} f_0 = 0\,, \tag{A.9}$$

where f_0 is a function with conformal weights $(0, 0)$ which does not depend on the undifferentiated ε [note that f_m has conformal weights $(m/2, 0)$ because $\varepsilon^m f_m$ has conformal weights $(0, 0)$; if $\overline{m}$ had been zero, we had arrived at (A.9) immediately]. The only way in which f_0 can depend nontrivially on the variables $\mathcal{D}^r\psi^\mu$ or $\mathcal{D}^{r+1}x^\mu$ $(r = 0, 1, \ldots)$ is through terms of the form $\eta\psi^\mu\psi^\nu f_{\mu\nu}(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}])$, $\eta\partial\varepsilon\psi^\mu f_\mu(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}])$, or $\eta\mathcal{D}x^\mu g_\mu(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}])$ [recall that the only variables (6.19) with negative L_0-weights are the undifferentiated η and ε and that η is an anticommuting variable]. (A.9) implies $f_{\mu\nu}(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}]) = 0$ and $f_\mu(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}]) = 0$. We conclude

$$f_0 = \eta\mathcal{D}x^\mu g_\mu(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}]) + h_0(y, x, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\partial\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}])\,. \tag{A.10}$$

We thus get the following intermediate result: without loss of generality we can assume

$$\omega = \sum_m \varepsilon^m h_m(y, x, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\partial\varepsilon, \eta], [\bar{\varepsilon}, \bar{\eta}]) + \eta\mathcal{D}x^\mu g_\mu(y, x, \partial\eta, C, [\bar{\mathcal{D}}x, \bar{\psi}], [\bar{\varepsilon}, \bar{\eta}])\,. \tag{A.11}$$

The only part of $\sigma_{0,1}$ which is active on such an ω is the part

$$\hat{\sigma}_{0,1} = \sum_{r\geq 0}\sum_{k=0}^{r} \binom{r}{k} (\partial^k\bar{\varepsilon}\,\bar{\mathcal{D}}^{r+1-k}x^\mu)\,\frac{\partial}{\partial(\bar{\mathcal{D}}^r\bar{\psi}^\mu)}\,.$$

Note that $\hat{\sigma}_{0,1}$ touches only the dependence on the variables $\bar{\mathcal{D}}^r\bar{\psi}^\mu$, $\bar{\mathcal{D}}^{r+1}x^\mu$ and $\bar{\partial}^r\bar{\varepsilon}$ $(r=0,1,\dots)$ and treats all other variables as contants. Hence, for ω as in (A.11), $\sigma_{0,1}\omega=0$ implies

$$\begin{aligned}\hat{\sigma}_{0,1}h_m(y,x,C,[\bar{\mathcal{D}}x,\bar{\psi}],[\partial\varepsilon,\eta],[\bar{\varepsilon},\bar{\eta}]) &= 0 \quad \forall m\,,\\ \hat{\sigma}_{0,1}g_\mu(y,x,\partial\eta,C,[\bar{\mathcal{D}}x,\bar{\psi}],[\bar{\varepsilon},\bar{\eta}]) &= 0\,.\end{aligned} \tag{A.12}$$

These equations are decomposed into pieces with definite degree in the undifferentiated $\bar{\varepsilon}$ and then analysed using the contracting homotopy

$$\bar{B}=\sum_{r\geq 0}(\bar{\mathcal{D}}^r\bar{\psi}^\mu)\,\frac{\partial}{\partial(\bar{\mathcal{D}}^{r+1}x^\mu)}\,.$$

By means of arguments analogous to those that have led to (A.11) we conclude that we can assume, without loss of generality,

$$\begin{aligned}h_m(y,x,C,[\bar{\mathcal{D}}x,\bar{\psi}],[\partial\varepsilon,\eta],[\bar{\varepsilon},\bar{\eta}]) &= \sum_q \bar{\varepsilon}^q h_{m,q}(y,x,C,[\partial\varepsilon,\eta],[\bar{\partial}\bar{\varepsilon},\bar{\eta}])+\\ &\quad +\bar{\eta}\bar{\mathcal{D}}x^\mu g_{m,\mu}(y,x,\bar{\partial}\bar{\eta},C,[\partial\varepsilon,\eta]),\\ g_\mu(y,x,\partial\eta,C,[\bar{\mathcal{D}}x,\bar{\psi}],[\bar{\varepsilon},\bar{\eta}]) &= \sum_q \bar{\varepsilon}^q h_{\mu,q}(y,x,\partial\eta,C,[\bar{\partial}\bar{\varepsilon},\bar{\eta}])+\\ &\quad +\bar{\eta}\bar{\mathcal{D}}x^\nu g_{\mu,\nu}(y,x,C,\partial\eta,\bar{\partial}\bar{\eta}).\end{aligned} \tag{A.13}$$

Since the $h_{m,q}$, $g_{m,\mu}$, $h_{\mu,q}$ and $g_{\mu,\nu}$ do not depend on the fermions, they are $\sigma_{0,1}$-invariant. We have thus proved that (A.1) implies

$$\begin{aligned}\omega &= h(y,x,C,[\varepsilon,\eta],[\bar{\varepsilon},\bar{\eta}])\\ &\quad +\eta\mathcal{D}x^\mu h_\mu(y,x,\partial\eta,C,[\bar{\varepsilon},\bar{\eta}])+\bar{\eta}\bar{\mathcal{D}}x^\mu\bar{h}_\mu(y,x,\bar{\partial}\bar{\eta},C,[\varepsilon,\eta])+\\ &\quad +\eta\bar{\eta}\mathcal{D}x^\mu\bar{\mathcal{D}}x^\nu h_{\mu\nu}(y,x,\partial\eta,\bar{\partial}\bar{\eta},C)+\sigma_{0,1}\hat{\omega}\,,\end{aligned} \tag{A.14}$$

where the functions on the right hand side (h, $\eta\mathcal{D}x^\mu h_\mu$, $\dots$, $\hat{\omega}$) are elements of $\mathcal{W}$. Note also that the sum on the right hand side is direct: no nonvanishing function $h+\eta\mathcal{D}x^\mu h_\mu+\bar{\eta}\bar{\mathcal{D}}x^\mu\bar{h}_\mu+\eta\bar{\eta}\mathcal{D}x^\mu\bar{\mathcal{D}}x^\nu h_{\mu\nu}$ is $\sigma_{0,1}$-exact because the various terms either do not contain variables $\mathcal{D}^{r+1}x^\mu$ or $\bar{\mathcal{D}}^{r+1}x^\mu$ at all, or they contain $\mathcal{D}x^\mu$ but no ε, or $\bar{\mathcal{D}}x^\mu$ but no $\bar{\varepsilon}$. Hence, our result characterizes $H(\sigma_{0,1},\mathcal{W})$ completely.

B. Derivation of (6.31)

We shall show that (6.30) implies (6.31). The proof is a case-by-case study for $g=0,\dots,4$. Since $\omega_{\overline{k}}$ does not depend on the fermions and has vanishing conformal weights, it can be assumed to contain only terms with even N_ε-degree and even $N_{\bar{\varepsilon}}$-degree. Hence, it does not depend on the supersymmetry ghosts if $g=0$ or $g=1$ which gives (6.31) in these cases. If $2\leq g\leq 4$ the assertion follows from

$$\sigma_{0,0}\omega_{\overline{k}}+\sigma_{0,1}\omega_{\overline{k}-1}=0\,, \tag{B.1}$$

which is the second equation in (6.29).

$\underline{g=2}$: Only $\omega_{\overline{k}=2}$ can depend on the supersymmetry ghosts. One has

$$\omega_{\overline{k}=2}=\varepsilon\partial\varepsilon a(X)+\bar{\varepsilon}\bar{\partial}\bar{\varepsilon}\bar{a}(X)\,,$$

where $a(X)$ and $\bar{a}(X)$ are functions of the undifferentiated x^μ and y^i. $\sigma_{0,0}\omega_{\overline{2}}$ contains for instance $\eta(\partial\varepsilon)^2a(X)$ and $\bar{\eta}(\bar{\partial}\bar{\varepsilon})^2\bar{a}(X)$ because $\sigma_{0,0}\varepsilon$ and $\sigma_{0,0}\bar{\varepsilon}$ contain $\eta\partial\varepsilon$ and $\bar{\eta}\bar{\partial}\bar{\varepsilon}$, respectively. If $a\neq 0$ or $\bar{a}\neq 0$, these terms are not $\sigma_{0,1}$-exact because they do not contain derivatives of an x^μ. We conclude that $a=0$ and $\bar{a}=0$ and thus that (6.31) holds for $g=2$.

$\underline{g=3}$: Again, only $\omega_{\overline{k}=2}$ can depend on the supersymmetry ghosts. The terms in $\omega_{\overline{k}=2}$ depending on ε or its derivatives are

$$\begin{aligned}&\eta\varepsilon\partial^2\varepsilon a(X)+\varepsilon\partial\varepsilon\partial\eta b(X)+\varepsilon\partial\varepsilon\bar{\partial}\bar{\eta}c(X)+\varepsilon\partial\varepsilon C^i d_i(X)+\\&+\bar{\eta}\bar{\mathcal{D}}x^\mu\varepsilon\partial\varepsilon e_\mu(X)+\eta(\partial\varepsilon)^2f(X)+\partial^2\eta\varepsilon^2g(X)\,.\end{aligned}\tag{B.2}$$

In addition there are analogous terms with $\bar{\varepsilon}$ or its derivatives. A straightforward calculation shows that (B.1) imposes

$$b=0\,,\quad c=0\,,\quad d_i=0\,,\quad e_\mu=\partial_\mu a\,,\quad f=a\,,\quad g=-\frac{1}{2}a\,,\tag{B.3}$$

where $a=a(X)$ is an arbitary function of the y^i and x^μ. Using (B.3) in (B.2), the latter becomes

$$\left[\eta\varepsilon\partial^2\varepsilon+\bar{\eta}\bar{\mathcal{D}}x^\mu\varepsilon\partial\varepsilon\partial_\mu+\eta(\partial\varepsilon)^2-\frac{1}{2}\partial^2\eta\varepsilon^2\right]a(X)=\sigma_0[\varepsilon\partial\varepsilon a(X)]+\sigma_{0,1}[\eta\partial\varepsilon\psi^\mu\partial_\mu a(X)]\,.\tag{B.4}$$

This shows that all terms containing ε or its derivatives can be removed from $\omega_{\overline{k}=2}$ by the redefinition $\omega'=\omega-\sigma_0[\varepsilon\partial\varepsilon a(X)+\eta\partial\varepsilon\psi^\mu\partial_\mu a(X)]$. Similarly one can remove all terms containing $\bar{\varepsilon}$ or its derivatives. Hence, without loss of generality one can assume $\omega_{\overline{k}=2}=0$ which implies (6.31) for $g=3$.

$\underline{g=4}$: Now $\omega_{\overline{k}=4}$ and ω_2 can depend on the supersymmetry ghosts. One has

$$\omega_{\overline{k}=4}=\varepsilon^3\partial^2\varepsilon a(X)+\varepsilon^2(\partial\varepsilon)^2b(X)+\bar{\varepsilon}^3\bar{\partial}^2\bar{\varepsilon}\bar{a}(X)+\bar{\varepsilon}^2(\bar{\partial}\bar{\varepsilon})^2\bar{b}(X)+\varepsilon\partial\varepsilon\bar{\varepsilon}\bar{\partial}\bar{\varepsilon}c(X)\,.$$

The fact that $\sigma_{0,0}\partial^2\varepsilon$ contains $-(1/2)\varepsilon\partial^3\eta$ implies $a=0$. Analogously one concludes $\bar{a}=0$. The fact that $\sigma_{0,0}\partial\varepsilon$ and $\sigma_{0,0}\bar{\partial}\bar{\varepsilon}$ contain $\eta\partial^2\varepsilon$ and $\bar{\eta}\bar{\partial}^2\bar{\varepsilon}$, respectively, implies $b=0$, $\bar{b}=0$ and $c=0$.

ω_2 is of the form $P^A(\text{ghosts},\mathcal{D}x^\mu,\bar{\mathcal{D}}x^\mu)a_A(X)$ where the P^A either depend on ε and its derivatives, or on $\bar{\varepsilon}$ and its derivatives. The complete list of polynomials P^A depending on ε and its derivatives is

$$\begin{aligned}&\eta\partial\eta\varepsilon\partial^2\varepsilon\,,\eta\partial\eta(\partial\varepsilon)^2\,,\partial^2\eta\partial\eta\varepsilon^2\,,\eta\partial^2\eta\varepsilon\partial\varepsilon\,,\eta\partial^3\eta\varepsilon^2\,,\\&\bar{\eta}\bar{\mathcal{D}}x^\mu\eta\varepsilon\partial^2\varepsilon\,,\bar{\eta}\bar{\mathcal{D}}x^\mu\eta(\partial\varepsilon)^2\,,\bar{\eta}\bar{\mathcal{D}}x^\mu\partial\eta\varepsilon\partial\varepsilon\,,\bar{\eta}\bar{\mathcal{D}}x^\mu\partial^2\eta\varepsilon^2\,,\\&\eta\bar{\partial}\bar{\eta}\varepsilon\partial^2\varepsilon\,,\eta\bar{\partial}\bar{\eta}(\partial\varepsilon)^2\,,\bar{\eta}\bar{\partial}^2\bar{\eta}\varepsilon\partial\varepsilon\,,\partial\eta\bar{\partial}\bar{\eta}\varepsilon\partial\varepsilon\,,\partial^2\eta\bar{\partial}\bar{\eta}\varepsilon^2\,,\bar{\eta}\bar{\mathcal{D}}x^\mu\bar{\partial}\bar{\eta}\varepsilon\partial\varepsilon\,,\\&\eta C^i\varepsilon\partial^2\varepsilon\,,\eta C^i(\partial\varepsilon)^2\,,\partial\eta C^i\varepsilon\partial\varepsilon\,,\partial^2\eta C^i\varepsilon^2\,,\bar{\partial}\bar{\eta}C^i\varepsilon\partial\varepsilon\,,\bar{\eta}\bar{\mathcal{D}}x^\mu C^i\varepsilon\partial\varepsilon\,,C^iC^j\varepsilon\partial\varepsilon\,.\end{aligned}$$

Starting with the terms

$$\varepsilon\partial^2\varepsilon\eta\partial\eta A_1(X) + (\partial\varepsilon)^2\eta\partial\eta B_1(X) + \varepsilon^2\partial\eta\partial^2\eta E_2(X) \tag{B.5}$$

one finds that (B.1) implies $A_1(X) = B_1(X) = 2E_2(X)$. Considering the terms

$$\begin{aligned} &\varepsilon\partial\varepsilon\eta\partial^2\eta B_5(X) + \varepsilon^2\eta\partial^3\eta E_1(X) + \varepsilon\partial^2\varepsilon\eta\bar{\eta}\bar{\mathcal{D}}x^\mu A_{4,\mu}(X) \\ &+(\partial\varepsilon)^2\eta\bar{\eta}\bar{\mathcal{D}}x^\mu B_{4,\mu}(X) + \varepsilon\partial\varepsilon\partial\eta\bar{\eta}\bar{\mathcal{D}}x^\mu C_{4,\mu}(X) + \varepsilon^2\partial^2\eta\bar{\eta}\bar{\mathcal{D}}x^\mu E_{6,\mu}(X)\,, \end{aligned} \tag{B.6}$$

one observes that the σ_0 transformation of these terms neither contain $\bar{\partial}^k\bar{\eta}$ or $\bar{\partial}^k\bar{\varepsilon}$ terms nor U(1) ghosts. Thus they have to fulfill (B.1) separately and one obtains

$$\begin{aligned} C_{4,\mu}(X) &= -\partial_\mu A_1(X) \\ B_{4,\mu}(X) &= -\partial_\mu B_5(X) + \partial_\mu A_1(X) - 2E_{6,\mu}(X) \\ A_{4,\mu}(X) &= -2\partial_\mu E_1(X) - 2E_{6,\mu}(X)\,. \end{aligned}$$

Eliminating the coefficients one finds that (B.5)+ (B.6) can be expressed by

$$\begin{aligned} &\sigma_0\left(\eta(\partial\varepsilon)^2(B_5(X) - A_1(X)) + \eta\varepsilon\partial^2\varepsilon E_1(X) + \varepsilon\partial\varepsilon\partial\eta A_1(X) - 2\varepsilon\partial\varepsilon\bar{\eta}\bar{\mathcal{D}}x^\mu E_{6,\mu}(X)\right) + \\ &+\sigma_{0,1}\left(-\eta\partial\eta\partial\varepsilon\psi^\mu\partial_\mu A_1(X) - 2\bar{\eta}\eta\partial\varepsilon\bar{\mathcal{D}}x^\mu\psi^\nu\partial_\nu E_{6,\mu} - \bar{\eta}\eta\partial\varepsilon\bar{\mathcal{D}}x^\rho\psi^\nu\Omega_{\rho\nu}{}^\mu E_{6,\mu}\right)\,, \end{aligned} \tag{B.7}$$

where we have used the on-shell equality (6.15). Next we consider the terms involving derivatives of $\bar{\eta}$

$$\begin{aligned} &\varepsilon\partial^2\varepsilon\eta\bar{\partial}\bar{\eta}A_2(X) + (\partial\varepsilon)^2\eta\bar{\partial}\bar{\eta}B_2(X) + \varepsilon\partial\varepsilon\bar{\eta}\bar{\partial}^2\bar{\eta}B_6(X) + \\ &+\varepsilon\partial\varepsilon\partial\eta\bar{\partial}\bar{\eta}B_7(X) + \varepsilon^2\partial^2\eta\bar{\partial}\bar{\eta}E_3(X) + \varepsilon\partial\varepsilon\bar{\partial}\bar{\eta}\bar{\eta}\bar{\mathcal{D}}x^\mu C_{5,\mu}(X)\,, \end{aligned} \tag{B.8}$$

which implies via (B.1)

$$\begin{aligned} &B_7(X) = 0\,, \qquad A_2(X) = B_6(X) = B_2(X) = -2E_3(X)\,, \\ &C_{5,\mu}(X) = -\partial_\mu A_2(X)\,. \end{aligned} \tag{B.9}$$

Thus (B.8) can be written as

$$\sigma_0\left(\varepsilon\partial\varepsilon\bar{\partial}\bar{\eta}A_2(X)\right) - \sigma_{0,1}\left(\partial\varepsilon\bar{\partial}\bar{\eta}\eta\psi^\mu\partial_\mu A_2(X)\right) \tag{B.10}$$

and thus be removed from ω_2. In the last step we consider contributions containing U(1) ghosts, i.e.

$$\begin{aligned} &\varepsilon\partial^2\varepsilon\eta C^i A_{3,i}(X) + (\partial\varepsilon)^2\eta C^i B_{3,i}(X) + \varepsilon\partial\varepsilon\partial\eta C^i B_{8,i}(X) + \varepsilon^2\partial^2\eta C^i E_{4,i}(X) + \\ &+\varepsilon\partial\varepsilon\bar{\partial}\bar{\eta}C^i B_{9,i}(X) + \varepsilon\partial\varepsilon C^i\bar{\eta}\bar{\mathcal{D}}x^\mu C_{6,\mu i}(X) + \varepsilon\partial\varepsilon C^i C^j B_{10,ij}(X)\,. \end{aligned} \tag{B.11}$$

(B.1) imposes $B_{10,ij}(X) = B_{9,i}(X) = B_{8,i}(X) = 0$. Furthermore we derive the conditions

$$A_{3,i}(X) = B_{3,i}(X) = -2E_{4,i}(X) \quad C_{6,\mu i}(X) = -\partial_\mu A_{3,i}(X)\,. \tag{B.12}$$

Using the on-shell equality (6.14), (B.11) can be written as

$$\sigma_0\left(\varepsilon\partial\varepsilon C^i A_{3,i}(X)\right)+\sigma_{0,1}\left(\partial\varepsilon C^i\eta\psi^\mu\partial_\mu A_{3,i}(X)-\partial\varepsilon\eta\bar{\eta}\psi^\mu\bar{\mathcal{D}}x^\nu(\Omega_{\mu\nu i}-\Omega_{\mu\nu}{}^\lambda G_{\lambda i})A_{3,i}(X)\right)\ . \tag{B.13}$$

Hence, as in the case $g=3$ one finds that (B.1) implies $\omega_2=\sigma_0(\dots)+\sigma_{0,1}(\dots)$ which implies (6.31) for $g=4$.

C. Analysis of Bianchi identities

In this appendix we summarize briefly the investigation of the Bianchi identities for two-dimensional supergravity coupled to Maxwell theory. The starting point is the structure equation

$$[\mathcal{D}_A,\mathcal{D}_B\}=-T_{AB}{}^C\mathcal{D}_C-R_{AB}\delta_L-F_{AB}{}^i\delta_i\,, \tag{C.1}$$

where $[\cdot,\cdot\}$ denotes the graded commutator, $\{\mathcal{D}_A\}=\{\mathcal{D}_a,\mathcal{D}_\alpha\}$ contains the covariant derivatives $\mathcal{D}_a$ and covariant supersymmetry transformations $\mathcal{D}_\alpha$, $\delta_L=(1/2)\varepsilon^{ab}l_{ab}$ is the Lorentz generator and δ_i are the U(1) generators (represented trivially in our case). The "torsions" $T_{AB}{}^C$, "curvatures" R_{AB} and "field strengths" $F_{AB}{}^i$ are generically field dependent and determined from the Bianchi identities implied by (C.1). Using the constraints (2.5) and (2.6) one obtains for the torsions

$$\begin{aligned}T_{\alpha\beta}{}^a&=2\mathrm{i}(\gamma^aC)_{\alpha\beta}\\ T_{a\beta}{}^\alpha&=\frac{1}{4}S(\gamma_a)_\beta{}^\alpha\\ T_{ab}{}^\alpha&=\frac{\mathrm{i}}{4}\varepsilon_{ab}(C\gamma_*)^{\alpha\beta}\mathcal{D}_\beta S\,,\end{aligned} \tag{C.2}$$

where S is the auxiliary scalar field of the gravitational multiplet. For the curvatures one obtains

$$\begin{aligned}R_{\alpha\beta}&=\mathrm{i}S(\gamma_*C)_{\alpha\beta}\\ R_{a\alpha}&=\frac{\mathrm{i}}{2}(\gamma_a\gamma_*)_\alpha{}^\beta\mathcal{D}_\beta S\\ R_{ab}&=\frac{1}{4}\varepsilon_{ab}(S^2+\mathcal{D}^2S),\end{aligned} \tag{C.3}$$

and the field strengths are given by

$$\begin{aligned}F_{\alpha\beta}{}^i&=2\mathrm{i}(\gamma_*C)_{\alpha\beta}\phi^i\\ F_{a\alpha}{}^i&=(\gamma_a)_\alpha{}^\beta\lambda^i_\beta\,.\end{aligned} \tag{C.4}$$

The supersymmetry transformations of λ^i_β and F^i_{ab} turn out to be

$$\begin{aligned}\mathcal{D}_\alpha\lambda^i_\beta&=\mathrm{i}(\gamma^a\gamma_*C)_{\alpha\beta}\mathcal{D}_a\phi^i+\frac{\mathrm{i}}{2}(\gamma_*C)_{\alpha\beta}\varepsilon^{ab}F^i_{ba}+\frac{\mathrm{i}}{2}(\gamma_*C)_{\alpha\beta}S\phi^i\\ \mathcal{D}_\alpha F^i_{ab}&=-(\gamma_b\mathcal{D}_a\lambda^i)_\alpha+(\gamma_a\mathcal{D}_b\lambda^i)_\alpha+\frac{1}{2}\varepsilon_{ab}\mathcal{D}_\alpha S\phi^i+\frac{1}{2}\varepsilon_{ab}S(\gamma_*)_\alpha{}^\delta\lambda^i_\delta\,.\end{aligned} \tag{C.5}$$

Introducing the corresponding connection 1-forms and proceeding along the lines of [14] one identifies the covariant derivatives $\mathcal{D}_a$ in terms of partial derivatives and connections, and the curvatures, field strengths and torsions with two lower Lorentz indices in terms of the connections and the other field strengths. Owing to the constraint $T_{ab}{}^c = 0$ this yields the expression (2.2) for the spin connection. Furthermore one obtains

$$F_{ab}{}^i = E_a{}^n E_b{}^m (\partial_n A^i_m - \partial_m A^i_n - (\chi_m \gamma_n \lambda^i) + (\chi_n \gamma_m \lambda^i) - 2\mathrm{i}(\chi_m \gamma_* C \chi_n)\phi^i)$$

and the expression for $T_{ab}{}^\alpha$ can be used to express the supersymmetry transformation of the auxiliary field S as

$$\mathcal{D}_\alpha S = 4\mathrm{i}(\gamma_* C)_{\alpha\beta}\varepsilon^{nm}\nabla_m \chi_n{}^\beta - \mathrm{i}(\gamma^m C)_{\alpha\beta}\chi_m{}^\beta S\,.$$

The full BRST transformations (2.1), (2.3) and (2.4) are then obtained by adding the Weyl transformations by hand and imposing $s^2 = 0$ on all fields. To achieve this in an off-shell setting, one introduces the super-Weyl symmetry on the gravitino and the gaugino and the local shift symmetry of the auxiliary field S.

D. BRST transformations of superconformal tensor fields

This appendix collects the BRST transformations of the superconformal tensor fields and corresponding ghost variables derived in section 3. The transformations of the undifferentiated fields read

$$\begin{aligned}
s\eta &= \eta\partial\eta - \varepsilon\varepsilon \\
s\bar\eta &= \bar\eta\bar\partial\bar\eta - \bar\varepsilon\bar\varepsilon \\
s\varepsilon &= \eta\partial\varepsilon - \frac{1}{2}\varepsilon\partial\eta \\
s\bar\varepsilon &= \bar\eta\bar\partial\bar\varepsilon - \frac{1}{2}\bar\varepsilon\bar\partial\bar\eta \\
sC^i &= \eta\bar\eta\mathcal{F}^i + \eta\bar\varepsilon\lambda^i + \bar\eta\varepsilon\bar\lambda^i + \varepsilon\bar\varepsilon\hat\phi^i \\
sX^M &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})X^M + \varepsilon\psi^M + \bar\varepsilon\bar\psi^M \\
s\psi^M &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\psi^M + \frac{1}{2}\partial\eta\psi^M + \varepsilon\mathcal{D}X^M - \bar\varepsilon\hat F^M \\
s\bar\psi^M &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\bar\psi^M + \frac{1}{2}\bar\partial\bar\eta\bar\psi^M + \bar\varepsilon\bar{\mathcal{D}}X^M + \varepsilon\hat F^M \\
s\hat F^M &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\hat F^M + \frac{1}{2}(\partial\eta + \bar\partial\bar\eta)\hat F^M + \varepsilon\mathcal{D}\bar\psi^M - \bar\varepsilon\bar{\mathcal{D}}\psi^M \\
s\hat\phi^i &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\hat\phi^i + \frac{1}{2}(\partial\eta + \bar\partial\bar\eta)\hat\phi^i + \varepsilon\lambda^i + \bar\varepsilon\bar\lambda^i \\
s\lambda^i &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\lambda^i + (\partial\eta + \frac{1}{2}\bar\partial\bar\eta)\lambda^i + \varepsilon\mathcal{D}\hat\phi^i + \bar\varepsilon\mathcal{F}^i + \partial\varepsilon\hat\phi^i \\
s\bar\lambda^i &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\bar\lambda^i + (\frac{1}{2}\partial\eta + \bar\partial\bar\eta)\bar\lambda^i + \bar\varepsilon\bar{\mathcal{D}}\hat\phi^i - \varepsilon\mathcal{F}^i + \bar\partial\bar\varepsilon\hat\phi^i \\
s\mathcal{F}^i &= (\eta\mathcal{D} + \bar\eta\bar{\mathcal{D}})\mathcal{F}^i + (\partial\eta + \bar\partial\bar\eta)\mathcal{F}^i - \varepsilon\mathcal{D}\bar\lambda^i + \bar\varepsilon\bar{\mathcal{D}}\lambda^i - \partial\varepsilon\bar\lambda^i + \bar\partial\bar\varepsilon\lambda^i
\end{aligned}$$

JHEP08(2001)023

The s-transformations of covariant $\mathcal{D}$ or $\bar{\mathcal{D}}$ derivatives (of first or higher order) of a field are obtained by applying $\mathcal{D}$'s and/or $\bar{\mathcal{D}}$'s to the transformations given above, using the rules $\mathcal{D}\eta = \partial\eta$, $\mathcal{D}\bar{\eta} = 0$, $\mathcal{D}\varepsilon = \partial\varepsilon$, $\mathcal{D}\bar{\varepsilon} = 0$ etc, as well as $[\mathcal{D}, \bar{\mathcal{D}}] = 0$. E.g., one gets

$$\begin{aligned}
s\mathcal{D}X^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\mathcal{D}X^M + \partial\eta\mathcal{D}X^M + \varepsilon\mathcal{D}\psi^M + \bar{\varepsilon}\mathcal{D}\bar{\psi}^M + \partial\varepsilon\psi^M \\
s\bar{\mathcal{D}}X^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\bar{\mathcal{D}}X^M + \bar{\partial}\bar{\eta}\bar{\mathcal{D}}X^M + \varepsilon\bar{\mathcal{D}}\psi^M + \bar{\varepsilon}\bar{\mathcal{D}}\bar{\psi}^M + \bar{\partial}\bar{\varepsilon}\bar{\psi}^M \\
s\mathcal{D}\bar{\mathcal{D}}X^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\mathcal{D}\bar{\mathcal{D}}X^M + (\partial\eta + \bar{\partial}\bar{\eta})\mathcal{D}\bar{\mathcal{D}}X^M + \varepsilon\mathcal{D}\bar{\mathcal{D}}\psi^M + \bar{\varepsilon}\mathcal{D}\bar{\mathcal{D}}\bar{\psi}^M + \\
&\quad + \partial\varepsilon\bar{\mathcal{D}}\psi^M + \bar{\partial}\bar{\varepsilon}\mathcal{D}\bar{\psi}^M \\
s\mathcal{D}\psi^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\mathcal{D}\psi^M + \frac{3}{2}\partial\eta\mathcal{D}\psi^M + \frac{1}{2}\partial^2\eta\psi^M + \varepsilon\mathcal{D}^2X^M + \partial\varepsilon\mathcal{D}X^M - \\
&\quad - \bar{\varepsilon}\mathcal{D}\hat{F}^M \\
s\bar{\mathcal{D}}\bar{\psi}^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\bar{\mathcal{D}}\bar{\psi}^M + \frac{3}{2}\bar{\partial}\bar{\eta}\bar{\mathcal{D}}\bar{\psi}^M + \frac{1}{2}\bar{\partial}^2\bar{\eta}\bar{\psi}^M + \bar{\varepsilon}\bar{\mathcal{D}}^2X^M + \bar{\partial}\bar{\varepsilon}\bar{\mathcal{D}}X^M + \\
&\quad + \varepsilon\bar{\mathcal{D}}\hat{F}^M \\
s\bar{\mathcal{D}}\psi^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\bar{\mathcal{D}}\psi^M + \frac{1}{2}\partial\eta\bar{\mathcal{D}}\psi^M + \bar{\partial}\bar{\eta}\bar{\mathcal{D}}\psi^M + \varepsilon\mathcal{D}\bar{\mathcal{D}}X^M - \bar{\partial}\bar{\varepsilon}\hat{F}^M - \\
&\quad - \bar{\varepsilon}\bar{\mathcal{D}}\hat{F}^M \\
s\mathcal{D}\bar{\psi}^M &= (\eta\mathcal{D} + \bar{\eta}\bar{\mathcal{D}})\mathcal{D}\bar{\psi}^M + \partial\eta\mathcal{D}\bar{\psi}^M + \frac{1}{2}\bar{\partial}\bar{\eta}\mathcal{D}\bar{\psi}^M + \bar{\varepsilon}\mathcal{D}\bar{\mathcal{D}}X^M + \partial\varepsilon\hat{F}^M + \\
&\quad + \varepsilon\mathcal{D}\hat{F}^M .
\end{aligned}$$

E. BRST transformations of superconformal antifields

In this appendix we present the full s transformations of the superconformal antifields associated with the matter and gauge multiplets, using the following notation:

$$\begin{aligned}
G_{MN} &:= H_{(MN)}(X) \\
D_i &:= D_i(X) \\
\Omega_{KNM} &:= \partial_K H_{MN}(X) - \partial_M H_{KN}(X) + \partial_N H_{KM}(X) \\
&= 2\Gamma_{KNM} - H_{KNM} \quad (H_{KNM} = 3\partial_{[K}H_{NM]}) \\
R_{KLMN} &:= \partial_M\partial_{[K}H_{L]N}(X) - \partial_N\partial_{[K}H_{L]M}(X) \\
&= \frac{1}{2}\left(\partial_K\Omega_{LMN} - \partial_L\Omega_{KMN}\right) = \frac{1}{2}\left(\partial_M\Omega_{KNL} - \partial_N\Omega_{KML}\right).
\end{aligned}$$

Ω_{KNM} and R_{KLMN} enjoy the following properties:

$$\begin{aligned}
\Omega_{KMN} + \Omega_{KNM} = \Omega_{MKN} + \Omega_{NKM} &= 2\partial_K G_{MN} \\
R_{KLMN} = -R_{LKMN} = -R_{KLNM} \, , &\quad \partial_{[J}R_{KL]MN} = 0 \, .
\end{aligned}$$

The full BRST transformations of the undifferentiated superconformal matter antifields are

$$\begin{aligned}
sF^*_M &= -\hat{\phi}^i\partial_M D_i + 2G_{MN}\hat{F}^N + \psi^K\bar{\psi}^N\Omega_{KNM} + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})F^*_M + \frac{1}{2}(\partial\eta+\bar{\partial}\bar{\eta})F^*_M - \\
&\quad -\varepsilon\bar{\psi}^*_M + \bar{\varepsilon}\psi^*_M \\
s\psi^*_M &= \bar{\lambda}^i\partial_M D_i + \bar{\psi}^N\hat{\phi}^i\partial_N\partial_M D_i + 2G_{MN}\bar{\mathcal{D}}\psi^N + \bar{\mathcal{D}}X^N\psi^K\Omega_{KNM} - \hat{F}^N\bar{\psi}^K\Omega_{MKN} - \\
&\quad -\psi^K\bar{\psi}^N\bar{\psi}^L R_{KMLN} + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\psi^*_M + (\frac{1}{2}\partial\eta+\bar{\partial}\bar{\eta})\psi^*_M + \varepsilon X^*_M + \bar{\varepsilon}\bar{\mathcal{D}}F^*_M + \bar{\partial}\bar{\varepsilon}F^*_M \\
s\bar{\psi}^*_M &= -\lambda^i\partial_M D_i - \psi^N\hat{\phi}^i\partial_N\partial_M D_i + 2G_{MN}\mathcal{D}\bar{\psi}^N + \mathcal{D}X^N\bar{\psi}^K\Omega_{NKM} + \hat{F}^N\psi^K\Omega_{KMN} + \\
&\quad +\psi^K\psi^L\bar{\psi}^N R_{LKMN} + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\bar{\psi}^*_M + (\partial\eta+\frac{1}{2}\bar{\partial}\bar{\eta})\bar{\psi}^*_M + \bar{\varepsilon}X^*_M - \varepsilon\mathcal{D}F^*_M - \partial\varepsilon F^*_M \\
sX^*_M &= -2G_{MN}\mathcal{D}\bar{\mathcal{D}}X^N - \mathcal{D}X^K\bar{\mathcal{D}}X^L\Omega_{KLM} + \hat{F}^K\hat{F}^L\Omega_{MKL} + \mathcal{D}\bar{\psi}^K\bar{\psi}^L\Omega_{MLK} - \\
&\quad -\psi^K\bar{\mathcal{D}}\psi^L\Omega_{KML} + \mathcal{D}X^N\bar{\psi}^K\bar{\psi}^L R_{NMLK} + \bar{\mathcal{D}}X^N\psi^K\psi^L R_{LKNM} + \\
&\quad +\hat{F}^N\psi^K\bar{\psi}^L\partial_M\Omega_{KLN} + \frac{1}{2}\psi^R\psi^K\bar{\psi}^N\bar{\psi}^L\partial_M R_{KRLN} + \mathcal{F}^i\partial_M D_i - \\
&\quad -(\psi^N\bar{\lambda}^i - \bar{\psi}^N\lambda^i + \hat{F}^N\hat{\phi}^i + \psi^N\bar{\psi}^K\hat{\phi}^i\partial_K)\partial_N\partial_M D_i + \\
&\quad +(\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})X^*_M + (\partial\eta+\bar{\partial}\bar{\eta})X^*_M + \varepsilon\mathcal{D}\psi^*_M + \bar{\varepsilon}\bar{\mathcal{D}}\bar{\psi}^*_M + \partial\varepsilon\psi^*_M + \bar{\partial}\bar{\varepsilon}\bar{\psi}^*_M\,.
\end{aligned}$$

The s transformation of the superconformal antifields for the gauge multiplet read

$$\begin{aligned}
s\lambda^*_i &= \bar{\psi}^M\partial_M D_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\lambda^*_i + \frac{1}{2}\bar{\partial}\bar{\eta}\lambda^*_i + \varepsilon\phi^*_i - \bar{\varepsilon}\bar{A}^*_i \\
s\bar{\lambda}^*_i &= -\psi^M\partial_M D_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\bar{\lambda}^*_i + \frac{1}{2}\partial\eta\bar{\lambda}^*_i + \bar{\varepsilon}\phi^*_i - \varepsilon A^*_i \\
s\phi^*_i &= -\hat{F}^M\partial_M D_i - \psi^M\bar{\psi}^N\partial_N\partial_M D_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\phi^*_i + \frac{1}{2}(\partial\eta+\bar{\partial}\bar{\eta})\phi^*_i + \\
&\quad +\varepsilon\mathcal{D}\lambda^*_i + \bar{\varepsilon}\bar{\mathcal{D}}\bar{\lambda}^*_i - \varepsilon\bar{\varepsilon}C^*_i \\
sA^*_i &= -\mathcal{D}X^M\partial_M D_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})A^*_i + \partial\eta A^*_i + \bar{\varepsilon}\mathcal{D}\lambda^*_i - \varepsilon\mathcal{D}\bar{\lambda}^*_i - \partial\varepsilon\bar{\lambda}^*_i - \bar{\varepsilon}\bar{\varepsilon}C^*_i \\
s\bar{A}^*_i &= \bar{\mathcal{D}}X^M\partial_M D_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})\bar{A}^*_i + \bar{\partial}\bar{\eta}\bar{A}^*_i + \varepsilon\bar{\mathcal{D}}\bar{\lambda}^*_i - \bar{\varepsilon}\bar{\mathcal{D}}\lambda^*_i - \bar{\partial}\bar{\varepsilon}\lambda^*_i - \varepsilon\varepsilon C^*_i \\
sC^*_i &= -\mathcal{D}\bar{A}^*_i - \bar{\mathcal{D}}A^*_i + (\eta\mathcal{D}+\bar{\eta}\bar{\mathcal{D}})C^*_i + (\partial\eta+\bar{\partial}\bar{\eta})C^*_i\,.
\end{aligned} \tag{E.1}$$

The BRST transformations of covariant derivatives of the covariant antifields (such as $s\mathcal{D}X^*_M$) are obtained from the above formulae by means of the rules described in appendix D.

References

[1] P. Ramond, *Dual theory for free fermions*, *Phys. Rev.* **D 3** (1971) 2415.

[2] A. Neveu and J.H. Schwarz, *Factorizable dual model of pions*, *Nucl. Phys.* **B 31** (1971) 86.

[3] A. Neveu and J.H. Schwarz, *Quark model of dual pions*, *Phys. Rev.* **D 4** (1971) 1109.

[4] S. Deser and B. Zumino, *A complete action for the spinning string*, *Phys. Lett.* **B 65** (1976) 369.

[5] L. Brink, P. Di Vecchia and P. Howe, *A locally supersymmetric and reparametrization invariant action for the spinning string*, *Phys. Lett.* **B 65** (1976) 471.

[6] F. Brandt, A. Kling and M. Kreuzer, *Actions and symmetries of NSR superstrings and D-strings*, *Phys. Lett.* **B 494** (2000) 155 [`hep-th/0006152`].

[7] E. Bergshoeff, S. Randjbar-Daemi, A. Salam, H. Sarmadi and E. Sezgin, *Locally supersymmetric sigma model with Wess-Zumino term in two-dimensions and critical dimensions for strings*, *Nucl. Phys.* **B 269** (1986) 77.

[8] F. Brandt, J. Gomis and J. Simón, *The rigid symmetries of bosonic D-strings*, *Phys. Lett.* **B 419** (1998) 148 [`hep-th/9707063`].

[9] F. Brandt, J. Gomis and J. Simón, *Cohomological analysis of bosonic D-strings*, *Nucl. Phys.* **B 523** (1998) 623 [`hep-th/9712125`].

[10] G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in gauge theories*, *Phys. Rep.* **338** (2000) 439 [`hep-th/0002245`].

[11] G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in Einstein Yang-Mills theory*, *Nucl. Phys.* **B 455** (1995) 357 [`hep-th/9505173`].

[12] F. Brandt, N. Dragon and M. Kreuzer, *The gravitational anomalies*, *Nucl. Phys.* **B 340** (1990) 187.

[13] N. Dragon, *BRS symmetry and cohomology*, `hep-th/9602163`.

[14] F. Brandt, *Local BRST cohomology and covariance*, *Comm. Math. Phys.* **190** (1997) 459 [`hep-th/9604025`].

[15] G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in the antifield formalism, 1. General theorems*, *Comm. Math. Phys.* **174** (1995) 57 [`hep-th/9405109`].

[16] G. Barnich and M. Henneaux, *Consistent couplings between fields with a gauge freedom and deformations of the master equation*, *Phys. Lett.* **B 311** (1993) 123 [`hep-th/9304057`].

[17] F. Brandt, W. Troost and A. Van Proeyen, *Background charges and consistent continuous deformations of 2d gravity theories*, *Phys. Lett.* **B 374** (1996) 31 [`hep-th/9510195`].

[18] O. Piguet and S. P. Sorella, *Algebraic renormalization: Perturbative renormalization, symmetries and anomalies* (Lecture notes in physics: m28), Springer, Berlin 1995.

[19] F. Brandt, W. Troost and A. Van Proeyen, *The BRST–antibracket cohomology of 2d gravity*, *Nucl. Phys.* **B 464** (1996) 353 [`hep-th/9509035`].

[20] F. Brandt, J. Gomis and J. Simón, *D-string on near horizon geometries and infinite conformal symmetry*, *Phys. Rev. Lett.* **81** (1998) 1770 [hep-th/9803196].

[21] L. Alvarez-Gaume and D.Z. Freedman, *Geometrical structure and ultraviolet finiteness in the supersymmetric sigma model*, *Comm. Math. Phys.* **80** (1981) 443.

[22] J. Gates, S. J., C.M. Hull and M. Rocek, *Twisted multiplets and new supersymmetric nonlinear sigma models*, *Nucl. Phys.* **B 248** (1984) 157.

[23] C.M. Hull and E. Witten, *Supersymmetric sigma models and the heterotic string*, *Phys. Lett.* **B 160** (1985) 398.

[24] A. Kling, *BRST cohomology of Dirichlet-Superstrings*, diploma thesis, Vienna 1998 (unpublished).

[25] P. S. Howe, *Super Weyl transformations in two-dimensions*, *J. Phys.* **A12** (1979) 393.

[26] F. Brandt, *Gauge covariant algebras and local BRST cohomology*, *Contemp. Math.* **219** (1999) 53–67 [hep-th/9711171].

[27] F. Brandt, *Jet coordinates for local BRST cohomology*, *Lett. Math. Phys.* **55** (2001) 149 [math-ph/0103006].

[28] F. Delduc and F. Gieres, *Beltrami differentials, conformal models and their supersymmetric generalizations*, *Class. and Quant. Grav.* **7** (1990) 1907.

[29] R. Grimm, *Left-right decomposition of two-dimensional superspace geometry and its BRS structure*, *Ann. Phys. (NY)* **200** (1990) 49.

[30] I.A. Batalin and G.A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, *Phys. Rev.* **D 28** (1983) 2567.

[31] I.A. Batalin and G.A. Vilkovisky, *Gauge algebra and quantization*, *Phys. Lett.* **B 102** (1981) 27.

[32] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press, Princeton 1992.

[33] J. Gomis, J. París and S. Samuel, *Antibracket, antifields and gauge theory quantization*, *Phys. Rep.* **259** (1995) 1 [hep-th/9412228].